

# REAL EXCHANGE RATES AND PRIMARY COMMODITY PRICES

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# INTRODUCTION

- Real exchange rates (RER) among large developed economies are:
  - very volatile.
  - very persistent.
  - notoriously hard to relate to fundamentals.
- The RER disconnect puzzle. Engel (1999), Obstfeld and Rogoff (2001).

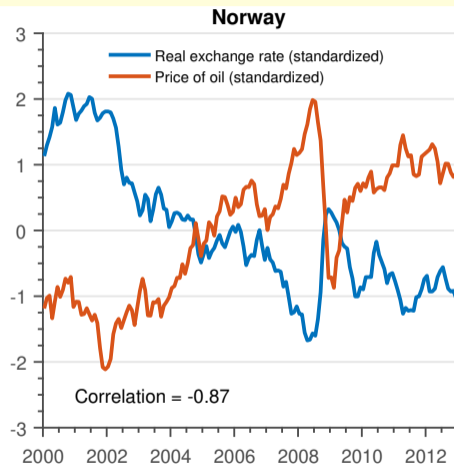
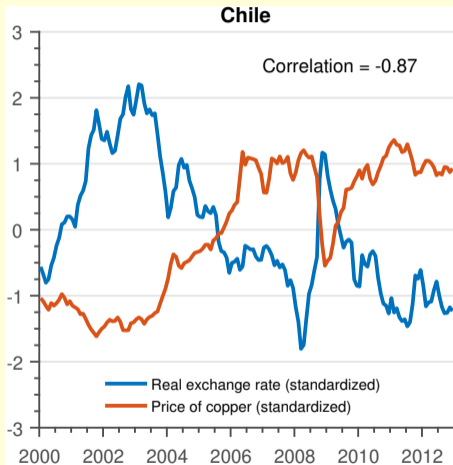
# INTRODUCTION

- Sticky prices and wages. (Chari, Kehoe, and McGrattan (2002))
- Segmented markets. (Itskhoki and Mukhin (2017))

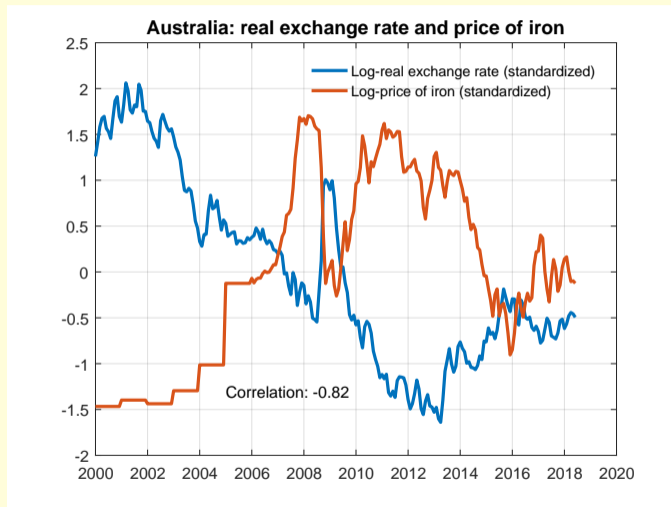
# INTRODUCTION

- No such disconnect in **small** open economies.
- Betts and Kehoe (2004), Chen and Rogoff (2003), and Hevia and Nicolini (2013), Ricci, Milesi-Ferreti and Lee (2013).
- Comovement between RER and primary commodity prices (PCP):
  - Chile (Copper).
  - Norway (Oil).
  - Australia (Iron, Coal).

# TWO SMALL OPEN ECONOMIES: CHILE AND NORWAY



# AUSTRALIA



# INTRODUCTION

- We show that PCP can solve the disconnect puzzle for developed economies.
- Existing literature for large developed economies ignores primary commodities.
- It should not.

# INTRODUCTION

- **Theoretical exercise:** incorporates primary commodity production.
  - Calibrate model to match volatility and persistence of PCPs (and other macro aggregates).
  - Equilibrium RERs almost as volatile and persistent as in the data.
  - RER and PCP highly correlated.
- **Data exercise:** substantial comovement between a few PCPs and RERs between Japan, UK and Germany against the US.
  - Model interpretation: shocks that move PCPs can account for a large fraction of the volatility and persistence of RERs.



# INTRODUCTION

## WHY PRIMARY COMMODITIES?

- The top 10 commodities account for 18% of world trade in 2012 (12% in 1990).
  - Only the **direct** measure.
- Traded in competitive markets, where the law of one price holds.
- Very volatile and persistent.

# INTRODUCTION

## POTENTIAL MECHANISM

- Primary commodities are at the bottom of the production chain.
- Movements in PCP change costs.
- Assume:
  - Input-output matrices of the two countries are different enough.
  - Countries produce different commodities.
- Then, PCP changes affect final good prices asymmetrically:
  - the RER ought to change.

# MODEL

- Two-country model plus: USA, Japan, and rest of the world (ROW).
- Three sectors: final goods (nontradable), intermediate goods, primary commodities.
  - Country 1 (USA) produces final good  $Y^1$ , intermediate good  $Q_1$ , and primary commodities  $X_1$  and  $X_3$ .
  - Country 2 (Japan) produces final good  $Y^2$ , intermediate good  $Q_2$ , and primary commodities  $X_2$  and  $X_3$ .
  - Country 3 (ROW) produces final good  $Y^3$ , and primary commodities  $X_1$ ,  $X_2$ , and  $X_3$ .
- Financial autarky: trade balance is zero in each period.
- No capital accumulation and labor inelastically supplied.
- Household preferences don't play any role.

## MODEL: PRODUCTION IN COUNTRY $i = 1, 2$ {USA, JPN}

- Final good production:

$$Y_t^i = Z_t^i (q_{1,t}^i)^{\alpha_1^i} (q_{2,t}^i)^{\alpha_2^i} (n_{y,t}^i)^{\alpha_3^i}$$

- Intermediate-good production:

$$Q_t^i = Z_t^i (x_{1,t}^i)^{\beta_1^i} (x_{2,t}^i)^{\beta_2^i} (x_{3,t}^i)^{\beta_3^i} (n_{q,t}^i)^{\beta_4^i}$$

- Primary-commodity production:

$$X_{i,t}^i = Z_t^i (e_i^i)^{1-\phi_i^i} (n_{x_i,t}^i)^{\phi_i^i}$$

$$X_{3,t}^i = Z_t^i (e_3^i)^{1-\phi_3^i} (n_{x_3,t}^i)^{\phi_3^i}$$

- $e_i^i$  and  $e_3^i$  are endowments of natural resources.
- There is also an endowment  $n^i$  of labor (inelastically supplied)

## MODEL: LOG - REAL EXCHANGE RATE

- Log RER:  $\tilde{\zeta}_t \equiv p_t^{y_1} + e_t^{1,2} - p_t^{y_2}$ .
- Prices equal marginal costs,
- which are Cobb-Douglas functions of input prices (linear in logs).
- Law of one price holds for primary commodities and intermediate goods.

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- Law of one price holds for primary commodities and intermediate goods.
- One representation of  $\tilde{\zeta}_t$  that holds in equilibrium is given by

$$\tilde{\zeta}_t = \gamma_{z_2} z_t^2 - \gamma_{z_1} z_t^1 + \gamma_{e_1} p_t^{e_1} + \gamma_{e_2} p_t^{e_2} + \gamma_{x_2} p_t^{x_2} + \gamma_{x_3} p_t^{x_3} + \left[ (\alpha_1^1 - \alpha_1^2) \beta_1^1 + (\alpha_2^1 - \alpha_2^2) \beta_1^2 + \frac{(\alpha_1^1 - \alpha_1^2) \beta_4^1 + \alpha_3^1}{\phi_1^1} \right] p_t^{x_1} \quad (1)$$

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$$\zeta_t = z_t^2 - z_t^1$$

if  $\alpha_1^1 = \alpha_1^2$  and  $\alpha_2^1 = \alpha_2^2$  and  $\alpha_3^1 = \alpha_3^2 = 0$ .

## MODEL: PRODUCTION IN ROW

- Device to generate volatile and persistent commodity prices.
- Final-good production:

$$Y_t^3 = \left(x_{1t}^3\right)^{\pi_1} \left(x_{2t}^3\right)^{\pi_2} \left(x_{3t}^3\right)^{\pi_3} \left(n_{y,t}^3\right)^{\pi_4}$$

- Stochastic endowment of the three commodities:  $X_{1t}^3, X_{2t}^3, X_{3t}^3$ 
  - Captures world contingencies that affect PCP such as the weather, natural disasters, monopolistic behavior by the OPEC, etc.
- **Shocks:**  $(Z_t^1, Z_t^2)$  and  $(X_{1t}^3, X_{2t}^3, X_{3t}^3)$  follow AR1 processes in logs.
  - Shocks to  $Z$  and  $X$  are orthogonal to each other.
  - But innovations in each block of shocks may be correlated.



## MODEL: CALIBRATION

- Factor shares: bilateral US-Japan input-output table from METI.
  - Primary commodities:
    - $X_1$ : crude petroleum and natural gas,
    - $X_2$ : fishing and seafood,
    - $X_3$ : the rest.
- Endowments: match relative GDPs in 1960–2014 and size of commodity sector in ROW GDP.
- Stochastic processes:
  - $(Z_1, Z_2)$ : match volatility, persistence, and cross-country correlations of real GDP.
  - $(X_{1t}^3, X_{2t}^3, X_{3t}^3)$ : match volatility, persistence, and correlations of PCPs.

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- **Models II and III**: negligible contribution of commodities to value added:  $\beta_{II} = \beta/100$  and  $\beta_{III} = \beta/10000$ .

# CALIBRATION OF STOCHASTIC PROCESSES

- Use SMM to calibrate the stochastic processes of TFP and ROW-endowment shocks.
- Minimize distance between model-generated moments and same moments using (HP-filtered) data

<b>Moments</b>	<b>Data</b>	<b>Baseline</b>
<b>standard deviation - RGDP USA (%)</b>	<b>1.3</b>	<b>1.3</b>
<b>standard deviation - RGDP JPN (%)</b>	<b>1.6</b>	<b>1.6</b>
<b>autocorrelation - RGDP USA</b>	<b>0.31</b>	<b>0.31</b>
<b>autocorrelation - RGDP JPN</b>	<b>0.18</b>	<b>0.18</b>
<b>correlation - USA and JPN RGDP</b>	<b>0.41</b>	<b>0.41</b>
standard deviation - price of oil (%)	66.7	86.7
standard deviation - price of fish (%)	35.5	29.4
standard deviation - price of aluminum (%)	31.5	31.7
autocorrelation - price of oil	0.92	0.99
autocorrelation - price of fish	0.76	0.79
autocorrelation - price of aluminum	0.84	0.99
correlation - price of oil and fish	0.29	0.22
correlation - price of oil and aluminium	-0.22	-0.72
correlation - price of fish and aluminium	0.37	0.27

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# MODEL RESULTS: NON-TARGETED MOMENTS

	Data	Baseline	Model II	Model III
<b>(1) Share of country GDP (%)</b>				
<b>United States</b>				
Final good $Y_1$	48.5	79.7	79.7	79.7
Intermediate good $Q_1$	47.9	18.0	20.3	20.3
Primary commodity $X_1$	1.2	0.8	0.01	0.00
Primary commodity $X_3$	2.4	1.5	0.01	0.00
<b>Japan</b>				
Final good $Y_2$	46.3	79.2	76.2	76.2
Intermediate good $Q_2$	49.9	20.5	23.8	23.8
Primary commodity $X_2$	0.4	0.0	0.0	0.0
Primary commodity $X_3$	3.4	0.2	0.0	0.0
<b>(2) Standard deviation of RER (%)</b>	37.0	27.8	5.9	1.5
<b>(3) Autocorrelation of RER</b>	0.96	0.99	0.95	0.23

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# EMPIRICAL METHODOLOGY

## GENERAL SETTING

- Primary commodity prices,  $\mathbf{p}_t^{X,USA} \in \mathbb{R}^m$ .
- State of the economy represented by a vector  $\omega_t \in \mathbb{R}^n$ .
- Equilibrium PCPs and RERs are functions of  $\omega_t$ . Linear approximation:

$$\zeta_t^{USA,UK} = \theta' \omega_t,$$

$$\mathbf{p}_t^{X,USA} = \Omega \omega_t.$$

- $R^2$  of the regression of  $\zeta_t^{USA,UK}$  on  $\mathbf{p}_t^{X,USA}$  measures how much of the variability of the RER can be accounted for by *fundamental shocks that affect PCPs*. [Details](#)

## MODEL RESULTS

Regression equation of the form

$$\tilde{\zeta}_t = \eta_0 + \eta_1 p_t^{x_1} + \eta_2 p_t^{x_2} + \eta_3 p_t^{x_3} + v_t$$

Coefficients of the OLS regressions using model simulated data

	$R^2$	$p^{x_1}$	$p^{x_2}$	$p^{x_3}$
Baseline regression	0.98	-0.007	-0.008	-0.866
Regression without $p_t^{x_3}$	0.24	0.156	-0.040	
Coefficients implied by RER equation	-	2.983	-2.612	-0.017



# EMPIRICAL RESULTS

## BENCHMARK REGRESSIONS

TABLE: Coefficients of determination  $R^2$

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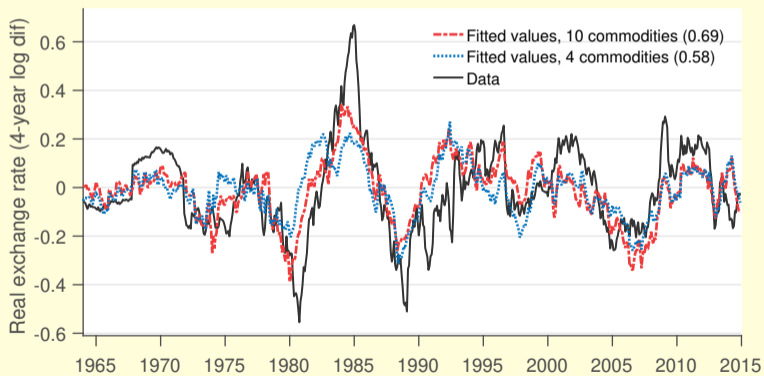
	<u>1960–2014</u>	<u>1960–1972</u>	<u>1973–1985</u>	<u>1986–1998</u>	<u>1999–2014</u>
<b>(a) 10 commodities, 4-year differences</b>					
United Kingdom	0.48	0.90	0.90	0.81	0.60
Germany	0.63	0.95	0.87	0.83	0.75
Japan	0.57	0.92	0.84	0.92	0.82
<b>(b) 4 commodities (best fit), 4-year differences</b>					
United Kingdom	0.33	0.72	0.82	0.63	0.58
Germany	0.56	0.84	0.87	0.81	0.74
Japan	0.48	0.88	0.76	0.86	0.80

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# EMPIRICAL RESULTS

## BENCHMARK REGRESSION: UNITED KINGDOM

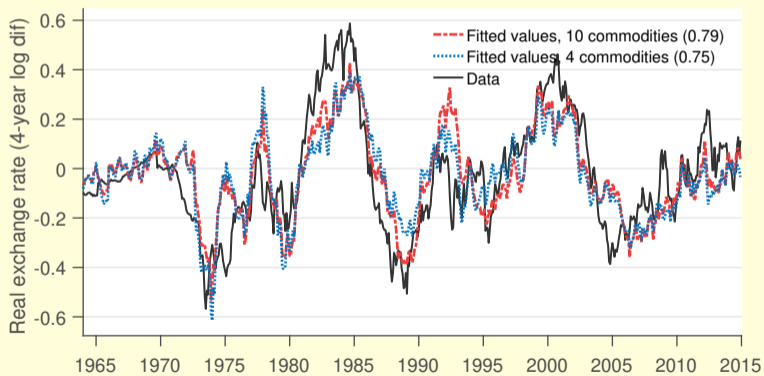
FIGURE: Real exchange rates and fitted values, four-year differences



# EMPIRICAL RESULTS

## BENCHMARK REGRESSION: GERMANY

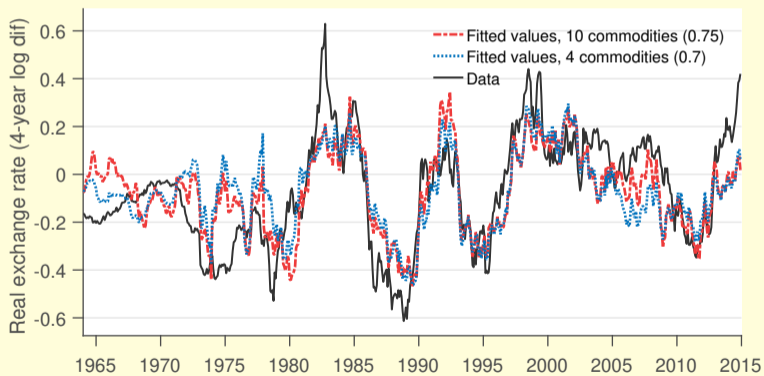
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# EMPIRICAL RESULTS

## BENCHMARK REGRESSION: JAPAN

FIGURE: Real exchange rates and fitted values, four-year differences



# EMPIRICAL RESULTS

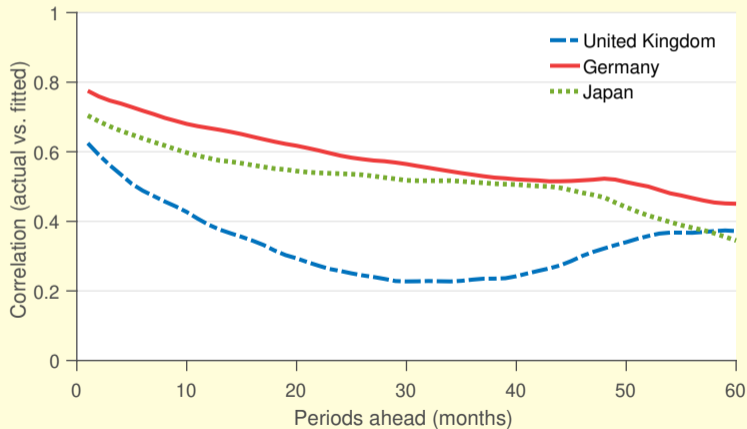
## OUT-OF-SAMPLE FIT

- Now we perform an out-of-sample exercise.
- Run the regression from Jan-1960 to Dec-1972. Choose the 4 commodities with the highest t-stats.
- Use data for those 4 PCP from Jan-1973 to Jan-1973+ $h$  and the estimated coefficients to fit value of the RER.
- Add one observation and repeat.
- Compare with data.

# EMPIRICAL RESULTS

## OUT-OF-SAMPLE FIT

FIGURE: Out-of-sample fit, four commodities, correlations as a function of  $h$



# LARGE VERSUS SMALL ECONOMIES

<b>Moments</b>	<b>Data</b>	<b>Benchmark</b>	<b>Model III</b>	<b>Model IV</b>
<b>share of US and Japan in world GDP (%)</b>	40.1	40.1	20.1	4.0
standard deviation - price of oil (%)	66.7	86.7	85.8	85.6
standard deviation - price of fish (%)	35.5	29.4	31.8	31.2
standard deviation - price of aluminum (%)	31.5	31.7	33.1	34.7
autocorrelation - price of oil	0.92	0.99	0.99	0.99
autocorrelation - price of fish	0.76	0.79	0.77	0.80
autocorrelation - price of aluminum	0.84	0.99	0.99	0.99
correlation - price of oil and fish	0.29	0.22	0.23	0.23
correlation - price of oil and aluminium	-0.22	-0.72	-0.71	-0.71
correlation - price of fish and aluminium	0.37	0.27	0.29	0.31
<b>standard deviation - RER (%)</b>	37.0	27.8	29.1	30.4
<b>autocorrelation - RER</b>	0.96	0.99	0.99	0.99

# MUSSA PUZZLE

## LINEAR TREND

<b>Moments</b>	<b>before 1973</b>		<b>after 1973</b>	
	<b>Data</b>	<b>Model</b>	<b>Data</b>	<b>Model</b>
standard deviation - price of oil (%)	12.7	12.7	51.3	52.0
standard deviation - price of fish (%)	26.1	26.0	36.6	36.9
standard deviation - price of aluminum (%)	3.9	4.8	22.9	22.3
autocorrelation - price of oil	0.64	0.74	0.88	0.93
autocorrelation - price of fish	0.35	0.30	0.80	0.67
autocorrelation - price of aluminum	0.63	0.74	0.61	0.91
correlation - price of oil and fish	0.59	0.46	0.72	0.07
correlation - price of oil and aluminium	-0.74	-0.98	0.48	-0.82
correlation - price of fish and aluminium	-0.50	-0.45	0.59	0.09
<b>standard deviation - RER (%)</b>	6.4	4.2	17.4	19.9
<b>autocorrelation - RER</b>	0.65	0.73	0.80	0.91



## CONCLUSIONS

- A simplifying – and somehow unfair – summary of the literature on exchange rates is that it has evolved according to a certain dichotomy.
- The role of trade in primary commodities has been explicitly modeled in studying developing economies.
- But models used to analyze large developed economies focuses on trade in differentiated final products exclusively and ignore trade in primary commodities.
- Maybe they should not.....

# APPENDIX

## LOG - REAL EXCHANGE RATE

$$\begin{aligned}\bar{\zeta}_t = & \left[ 1 - (\alpha_2^1 - \alpha_2^2) + \frac{(\alpha_2^1 - \alpha_2^2) \beta_4^2 + \alpha_3^2}{\phi_2^2} \right] z_t^2 \\ & - \left[ 1 + (\alpha_1^1 - \alpha_1^2) - \frac{(\alpha_1^1 - \alpha_1^2) \beta_4^1 + \alpha_3^1}{\phi_1^1} \right] z_t^1 \\ & + \left[ (\alpha_1^1 - \alpha_1^2) \beta_1^1 + (\alpha_2^1 - \alpha_2^2) \beta_1^2 + \frac{(\alpha_1^1 - \alpha_1^2) \beta_4^1 + \alpha_3^1}{\phi_1^1} \right] p_t^{x_1} \\ & + \left[ (\alpha_1^1 - \alpha_1^2) \beta_2^1 + (\alpha_2^1 - \alpha_2^2) \beta_2^2 + \frac{(\alpha_2^1 - \alpha_2^2) \beta_4^2 + \alpha_3^2}{\phi_2^2} \right] p_t^{x_2} \\ & + \left[ (\alpha_1^1 - \alpha_1^2) \beta_3^1 + (\alpha_2^1 - \alpha_2^2) \beta_3^2 \right] p_t^{x_3} \\ & - \left[ (\alpha_1^1 - \alpha_1^2) \beta_4^1 + \alpha_3^1 \right] \frac{1 - \phi_1^1}{\phi_1^1} p_{1,t}^{e_1} - \left[ (\alpha_2^1 - \alpha_2^2) \beta_4^2 - \alpha_3^2 \right] \frac{1 - \phi_2^2}{\phi_2^2} p_{2,t}^{e_2}\end{aligned}$$

# CALIBRATION: FACTOR SHARES - BILATERAL IO TABLE FROM METI

	Country 1 (USA)	Country 2 (JPN)
<b>Final good</b>		
intermediate good $Q_1$	$\alpha_1^1 = 20.2$	$\alpha_1^2 = 0.3$
intermediate good $Q_2$	$\alpha_2^1 = 0.1$	$\alpha_2^2 = 23.5$
labor $n_y$	$\alpha_3^1 = 79.7$	$\alpha_3^2 = 76.2$
<b>Intermediate good</b>		
primary commodity $X_1$	$\beta_1^1 = 6.5$	$\beta_1^2 = 5.9$
primary commodity $X_2$	$\beta_2^1 = 0.0$	$\beta_2^2 = 0.1$
primary commodity $X_3$	$\beta_3^1 = 4.7$	$\beta_3^2 = 11.5$
labor $n_q$	$\beta_4^1 = 88.8$	$\beta_4^2 = 82.5$
<b>Primary commodity <math>X_i</math></b>		
labor $n_{x_i}^i$	$\phi_1^1 = 33$	$\phi_2^2 = 27$
natural resource $e_j^i$	$1 - \phi_1^1 = 67$	$1 - \phi_2^2 = 63$
<b>Primary commodity <math>X_3</math></b>		
labor $n_{x_3}^i$	$\phi_1^1 = 52$	$\phi_3^2 = 34$
natural resource $e_3^i$	$1 - \phi_3^1 = 48$	$1 - \phi_3^2 = 66$

## CALIBRATION: FACTOR SHARES ROW

- We choose the share of labor in ROW to match share of agriculture and mining sector in ROW GDP in 2005.
- Then we choose shares of primary commodities according to their respective shares in total world trade in primary commodities.

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	<b>Rest of the world</b>
<b>Final good</b>	
$X_1$ : crude petroleum and natural gas	$\pi_1 = 5.8$
$X_2$ : fishing and seafood	$\pi_2 = 0.5$
$X_3$ : rest of commodities	$\pi_3 = 3.5$
$n_y^3$ : labor	$\pi_4 = 90.2$

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## CALIBRATION: RELATIVE SIZE OF THE COUNTRIES

- Normalize size of country 1 (USA) by setting  $n_1 = e_1^1 = e_3^1 = 1$ .
- Choose size of countries 2 (JPN) and 3 (ROW) to match average shares in world GDP from 1960 to 2014.

### Relative sizes in steady state

	Parameters	Share of world GDP (%)	
		Data	Model
Country 1 (USA)	$n_1 = e_1^1 = e_3^1 = 1$	30	30
Country 2 (Japan)	$n_2 = e_2^2 = e_3^2 = 0.33$	10	10
Country 3 (ROW)	$X_1^3 = 2.3; X_2^3 = 1.04; X_3^3 = 0.32$	60	60

# CALIBRATION OF STOCHASTIC PROCESSES

Parameter	Values	Parameter	Values
$100 \times \sigma \left( \varepsilon_t^{Z_1} \right)$	1.0	$100 \times \sigma \left( \varepsilon_t^{X_3^3} \right)$	1.8
$100 \times \sigma \left( \varepsilon_t^{Z_2} \right)$	1.2	$\rho^{X_1^3}$	0.99
$\rho^{Z_1}$	0.29	$\rho^{X_2^3}$	0.00
$\rho^{Z_2}$	0.17	$\rho^{X_3^3}$	0.99
$\rho \left( \varepsilon_t^{Z_1}, \varepsilon_t^{Z_2} \right)$	0.41	$\rho \left( \varepsilon_t^{X_1^3}, \varepsilon_t^{X_2^3} \right)$	-0.00
$100 \times \sigma \left( \varepsilon_t^{X_1^3} \right)$	6.6	$\rho \left( \varepsilon_t^{X_1^3}, \varepsilon_t^{X_3^3} \right)$	-0.77
$100 \times \sigma \left( \varepsilon_t^{X_2^3} \right)$	13.5	$\rho \left( \varepsilon_t^{X_2^3}, \varepsilon_t^{X_3^3} \right)$	0.08

# EMPIRICAL METHODOLOGY

## GENERAL SETTING

- Primary commodity prices,  $\mathbf{p}_t^{X,USA} \in \mathbb{R}^m$ .
- State of the economy represented by a vector  $\omega_t \in \mathbb{R}^n$ .
- Equilibrium PCPs and RERs are functions of the state variables. Using a linear approximation (if necessary):

$$\zeta_t^{USA,UK} = \theta' \omega_t,$$

$$\mathbf{p}_t^{X,USA} = \Omega \omega_t.$$

- Treat  $\omega_t$  as unobservable and interpret the state variables as orthogonal with an identity covariance matrix.



# EMPIRICAL METHODOLOGY

## GENERAL SETTING

- Consider the projection

$$\text{Proj} \left( \zeta_t^{USA,UK} \mid \mathbf{p}_t^{X,USA} \right) = \boldsymbol{\eta}' \mathbf{p}_t^{X,USA}$$

$$\boldsymbol{\eta}' = (\boldsymbol{\theta}' \boldsymbol{\Omega}') (\boldsymbol{\Omega} \boldsymbol{\Omega}')^{-1}$$

- The projection decomposes the RER into two orthogonal components:

$$\zeta_t^{USA,UK} = \boldsymbol{\eta}' \boldsymbol{\Omega} \omega_t + (\boldsymbol{\theta}' - \boldsymbol{\eta}' \boldsymbol{\Omega}) \omega_t$$

- $R^2$  of the projection:

$$R^2 = \frac{\boldsymbol{\eta}' \boldsymbol{\Omega} \boldsymbol{\Omega}' \boldsymbol{\eta}}{\boldsymbol{\theta}' \boldsymbol{\theta}}$$

- $R^2$  measures how much of the variability of the RER can be accounted for by *fundamental shocks that affect PCPs*.

# EMPIRICAL METHODOLOGY

## GENERAL SETTING

- Divide the state variables in two sets as  $\omega_t = [\omega'_{1t} \ \omega'_{2t}]'$ , so that

$$\tilde{\zeta}_t^{USA,UK} = \theta'_1 \omega_{1t} + \theta'_2 \omega_{2t}$$

$$\mathbf{p}_t^{X,USA} = \Omega_1 \omega_{1t} + \Omega_2 \omega_{2t}.$$

- Sufficient condition for the  $R^2$  of the regression to be zero:

$$\theta_1 = 0 \text{ and } \Omega_2 = 0$$

- Implies an equilibrium with a block-recursive structure in the model.
  - State variables that determine the RER ( $\omega_{2,t}$ ) are different from and orthogonal ( $\Omega_2 = 0$ ) to those that determine commodity prices.

# EMPIRICAL RESULTS: ROBUSTNESS

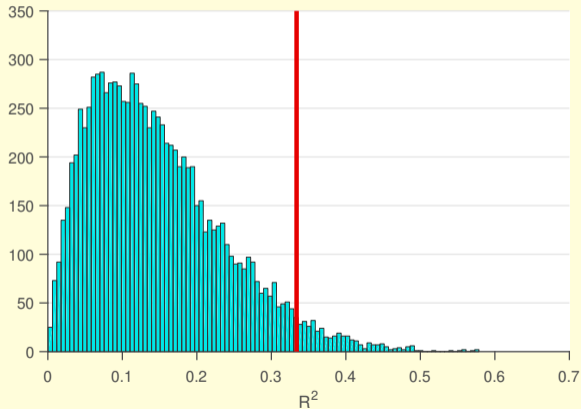
ARE THE RESULTS SPURIOUS?

- Estimate time series process for each RER.
- Estimate a VAR for the 10 PCP.
- Generate artificial data using Montecarlo under the null of orthogonality and reproduce the regressions.
- $R^2 \rightarrow 0$  as the sample size goes to  $\infty$ .
- Get 10.000 samples of size 660 and compute the distribution of the  $R^2$ .

# EMPIRICAL RESULTS: ROBUSTNESS

UNITED KINGDOM

FIGURE: Small sample distribution of the  $R^2$  over the period 1960–2014

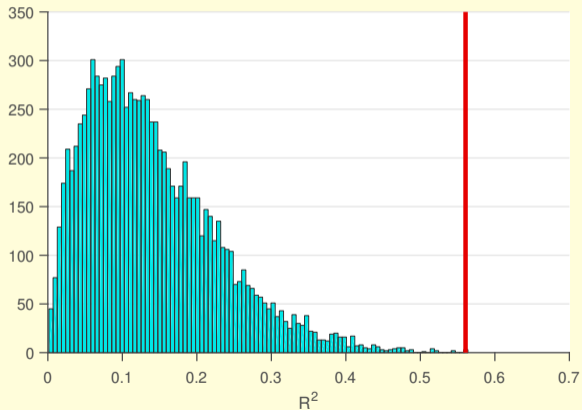


$$\Pr(R^2 > 0.33) = 0.037.$$

# EMPIRICAL RESULTS: ROBUSTNESS

GERMANY

FIGURE: Small sample distribution of the  $R^2$  over the period 1960–2014

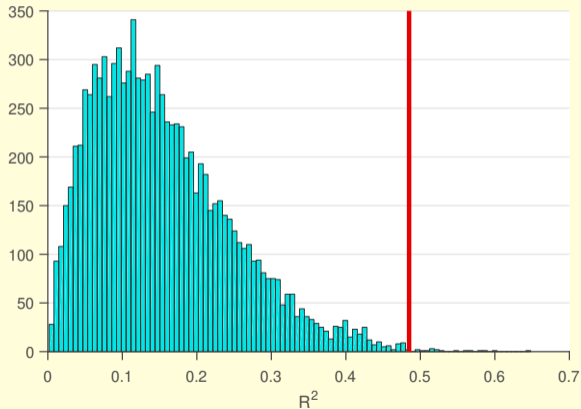


$$\Pr(R^2 > 0.56) = 0.000.$$

# EMPIRICAL RESULTS: ROBUSTNESS

JAPAN

FIGURE: Small sample distribution of the  $R^2$  over the period 1960–2014



$$\Pr(R^2 > 0.48) = 0.003.$$

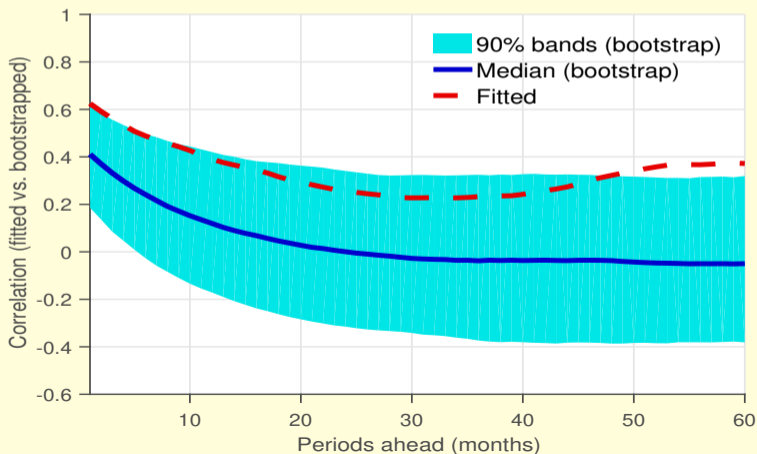
Bootstrapped distributions of  $R^2$  under the null hypothesis of orthogonality

	$\hat{R}^2$	Percentiles distribution of $R^2$				$\Pr(R^2 \geq \hat{R}^2)$
		Median	75	90	95	
<b>United Kingdom</b>						
1960-2014	0.33	0.13	0.20	0.27	0.31	0.037
1960-1972	0.72	0.52	0.66	0.75	0.80	0.143
1973-1985	0.82	0.37	0.52	0.64	0.70	0.004
1986-1998	0.63	0.37	0.50	0.61	0.67	0.077
1999-2014	0.58	0.29	0.41	0.53	0.59	0.059
<b>Germany</b>						
1960-2014	0.56	0.13	0.19	0.26	0.31	0.000
1960-1972	0.84	0.56	0.69	0.79	0.83	0.032
1973-1985	0.87	0.49	0.63	0.73	0.78	0.005
1986-1998	0.81	0.40	0.54	0.65	0.71	0.007
1999-2014	0.74	0.30	0.43	0.55	0.61	0.007
<b>Japan</b>						
1960-2014	0.48	0.14	0.21	0.29	0.34	0.003
1960-1972	0.88	0.59	0.72	0.81	0.85	0.022
1973-1985	0.76	0.46	0.60	0.70	0.75	0.045
1986-1998	0.86	0.41	0.55	0.66	0.71	0.001
1999-2014	0.80	0.33	0.46	0.57	0.63	0.002

# EMPIRICAL RESULTS: ROBUSTNESS

UNITED KINGDOM

FIGURE: Fitted correlations and (Montecarlo) error bands under the null hypothesis of orthogonality

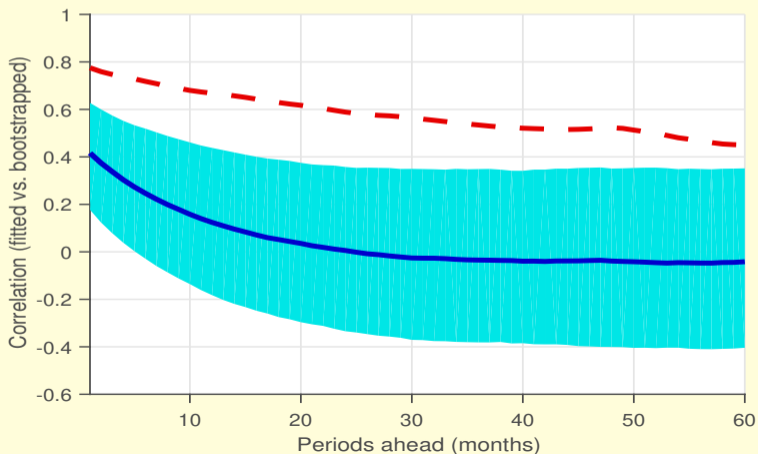




# EMPIRICAL RESULTS: ROBUSTNESS

GERMANY

FIGURE: Fitted correlations and (Montecarlo) error bands under the null hypothesis of orthogonality



# EMPIRICAL RESULTS: ROBUSTNESS

JAPAN

FIGURE: Fitted correlations and (Montecarlo) error bands under the null hypothesis of orthogonality

