Discussion of "Self-fulfilling Debt Crises, Revisited" by Aguiar, Chatterjee, Cole, Stangebye

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Introduction

- Were debt crises such as Mexico 1994–95 and Europe 2010-12 self-fulfilling?
- Theory of self-fulfilling debt crisis with short-term debt: Cole and Kehoe (2000) and Calvo (1988)
- Questions:
  - Are self-fulfilling crises quantitatively relevant?
  - What are their characteristics?
- This paper:
  - Quantitative model with rollover risk (Cole and Kehoe, 2000)
  - **New:** shocks between auction of new bonds and decision to default on old debt.
  - Results: higher default frequency and higher variation in spreads .

Mechanism in a two-period model

- Preferences:

$$U = \mathbb{E}\left[c_1 + \beta c_2\right]$$

- Endowment y in $t = 1, 2$ in case of no default.
- One-period bond traded with risk-neutral lenders ($\beta^L > \beta$).
- Option to default:
  - $y^d < y$ (output loss).
  - in $t = 1$, utility (or endowment) shock $\sigma\varepsilon$, $\varepsilon \sim U\left[0, 1\right]$.
- Timing in period $t = 1$ (rollover risk):
  1. Government auctions new bond.
  2. Shock ε is realized.
  3. Default decision on old debt (also implies default on new debt).

Mechanism in a two-period model

Moving backwards

- In $t = 2$:
  - Default if $y^d > y - a_2$.
- Eaton-Gersovitz price in $t = 1$:

$$q^{EG}(a_2) = \begin{cases} \beta^L, & \text{if } a_2 \leq y - y^d \\ 0, & \text{if } a_2 > y - y^d \end{cases}$$

Mechanism in a two-period model

Price of debt q

- Default decision in $t = 1$ after auction and shock realization ε:

$$\begin{aligned} V^r(a_1, a_2, q) &= (y - a_1 + q a_2) + \beta (y - a_2), \\ V^d(\varepsilon) &= (y^d + \sigma\varepsilon) + \beta y^d. \end{aligned}$$

- Zero-profit condition:

$$q = \underbrace{\Pr\left(\sigma\varepsilon \leq (1+\beta)\left(y - y^d\right) - a_1 + (q - \beta) a_2\right)}_{\text{rollover risk}} \times q^{EG}(a_2)$$

- Multiplicity:
  - Low $q \implies$ repayment less likely $\implies$ low q.
  - High $q \implies$ repayment more likely $\implies$ high q.

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- If government kept revenues from auction after default:
  - No multiplicity, price is unique.

Mechanism in a two-period model

Price correspondence $Q(a_1, a_2)$

- Possible prices: β^L, zero, interior price.

$$\beta^L \in Q(a_1, a_2) \text{ if } \begin{cases} \sigma \leq (1+\beta)\left(y - y^d\right) - a_1 + \left(\beta^L - \beta\right) a_2, & \text{and} \\ a_2 \leq y - y^d \end{cases}$$

$$0 \in Q(a_1, a_2) \text{ if } \begin{cases} 0 > (1+\beta)\left(y - y^d\right) - a_1 - \beta a_2, & \text{or} \\ a_2 > y - y^d \end{cases}$$

Interior price characterized by the threshold $\overline{\varepsilon}(a_1, a_2)$:

$$\overline{\varepsilon}(a_1, a_2) = \frac{a_1 + \beta a_2 - (1+\beta)\left(y - y^d\right)}{\beta^L a_2 - \sigma}$$

$$q = \underbrace{\overline{\varepsilon}(a_1, a_2)}_{F(\overline{\varepsilon}(a_1, a_2))} \beta^L$$

- For given (a_1, a_2), there is either one or three prices in $Q(a_1, a_2)$.

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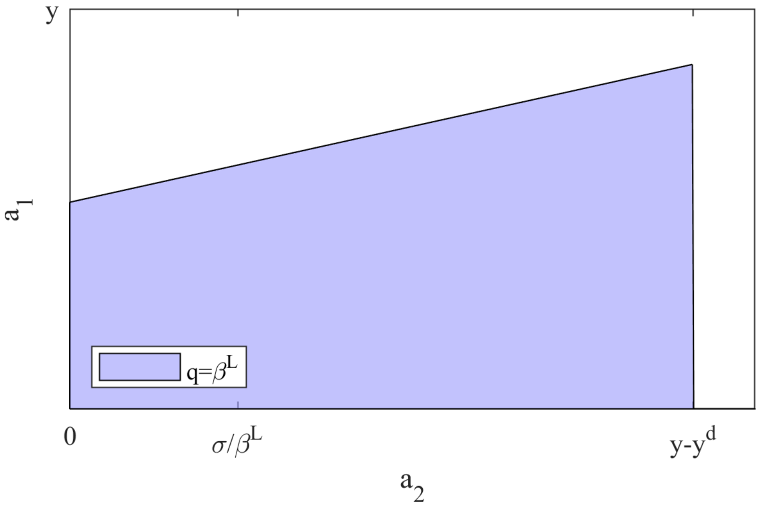
FIGURE: Multiplicity region: $\sigma > 0$



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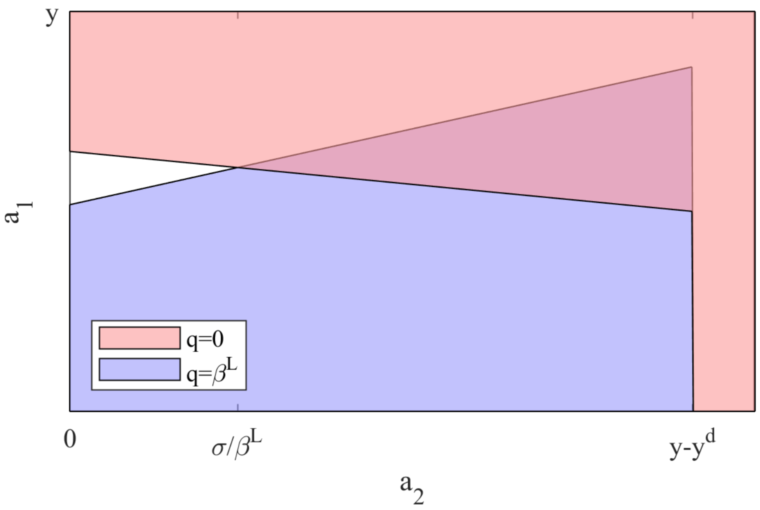

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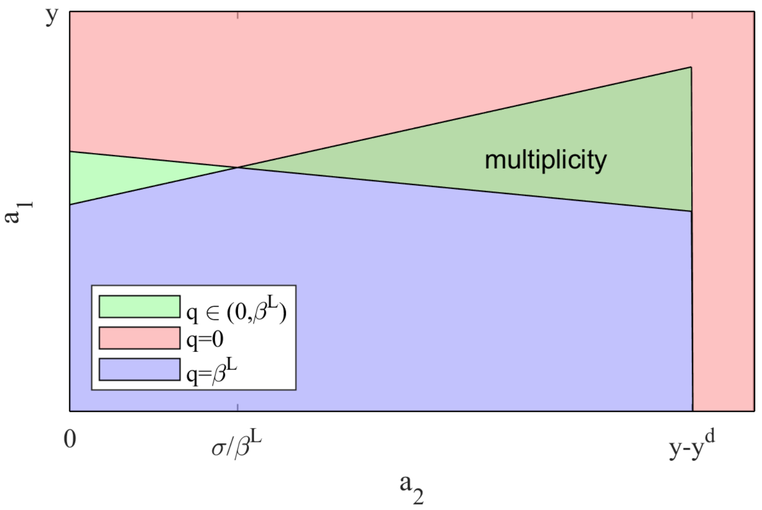

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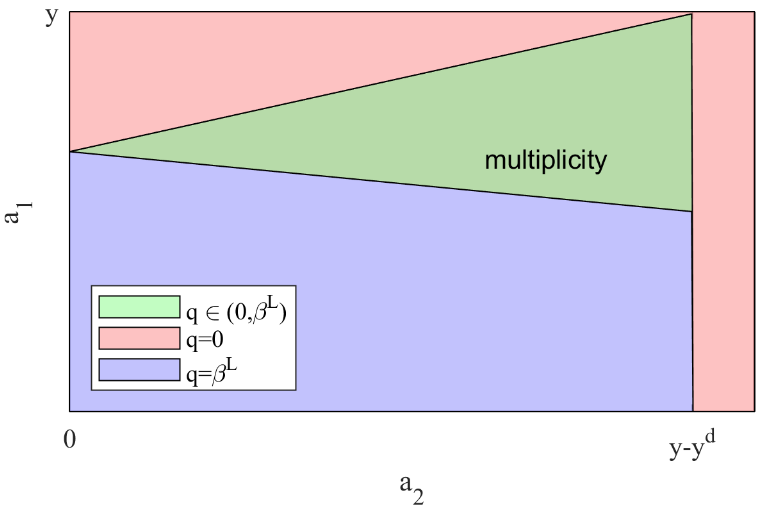

Case $\sigma = 0$

- Default decision depends on

$$V^r(a_1, a_2, q) - V^d = (1+\beta)\, y - a_1 + (q - \beta)\, a_2.$$

- If $V^r\left(a_1, a_2, \beta^L\right) > V^d$ and $V^r(a_1, a_2, 0) < V^d$, then there is $q^* \in (0, \beta^L)$ such that $V^r(a_1, a_2, q^*) = V^d$.

- Equivalent to Cole and Kehoe (2000) with randomization over default decision (mixed strategies).

- ε is simply another sunspot.

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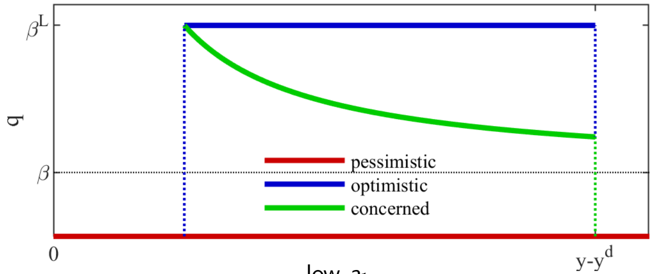
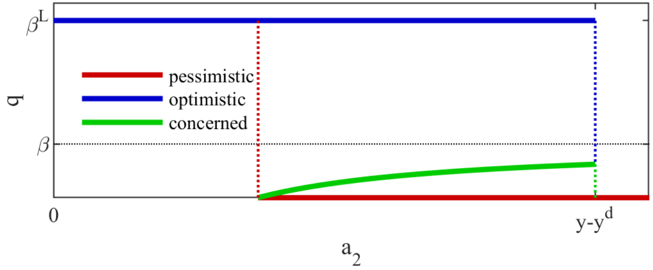
high a_1



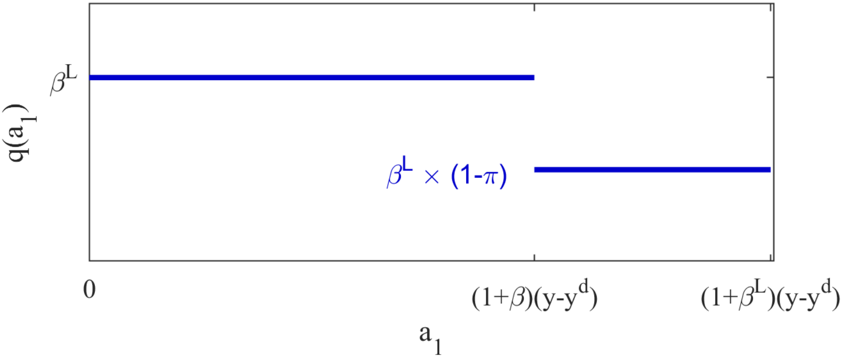
low a_1



$$V^r(a_1, a_2, q) = (1+\beta)y - a_1 + (q-\beta)a_2$$

Austerity vs Gambling for Redemption

Figure: Choosing a_1



- Government might choose to reduce debt to avoid lower price (Cole and Kehoe, 2000).
- Or not, due to impatience and/or consumption smoothing + hope of good endowment realization tomorrow (Conesa and Kehoe, 2017).

Similar mechanism without rollover risk

Ayres, Navarro, Nicolini, and Teles (2017)

- Restriction on strategy space:
  - borrow b today, pay $bR(b)$ tomorrow, instead of
  - borrow $q(a)a$ today, pay a tomorrow.
- Bimodal distribution in period 2:

$$y_2 = \begin{cases} y^l, & \text{probability } p \\ y^h, & \text{probability } (1-p) \end{cases}$$

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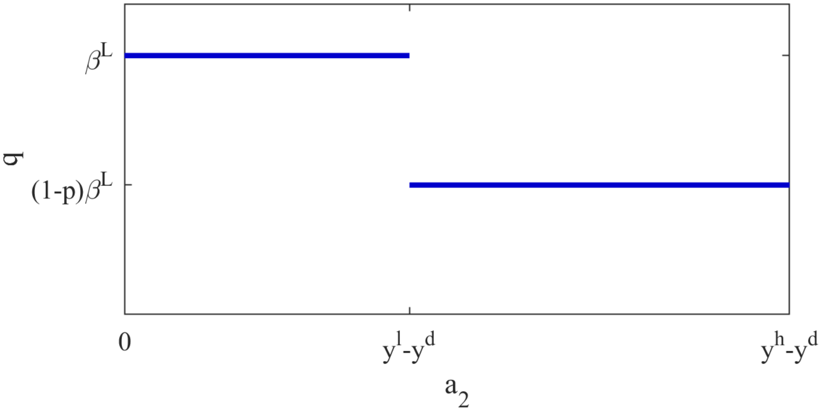
Figure: Price schedule



- Uniqueness: choosing a_2 implies choosing default probability.

Similar mechanism without rollover risk

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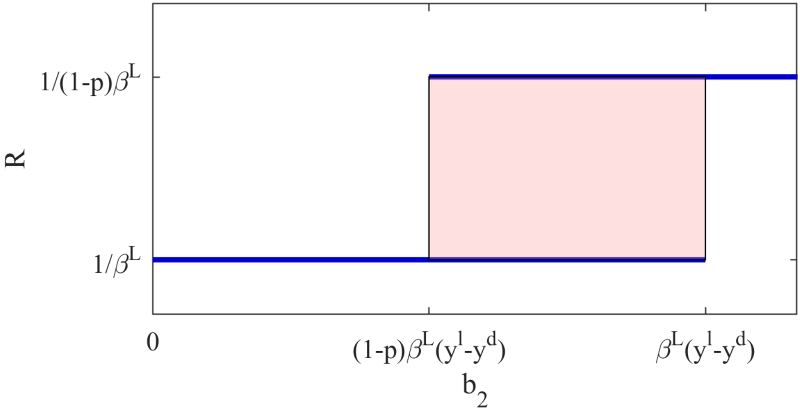
Figure: Interest rate correspondence



- Crisis today, higher probability of default tomorrow.

Other assumptions to consider

- Calvo + Cole and Kehoe.
  - Assumptions on different things:
    - Timing of auction and repayment (Cole and Kehoe, 2000).
    - Strategy space (Calvo, 1988).
  - Ayres and Paluszynski (2020) combine both:
    - No need for bimodal distribution, rollover risk generates multiplicity *a la Calvo*.
    - Expectation of rollover crisis in $t+1 \implies$ high $R \implies$ higher rollover risk in $t+1$.
- Government as price taker:
  - Lorenzoni and Werning (2018): government cannot commit to re-issue in the same period.
  - Ayres, Navarro, Nicolini, Teles (2018): lenders move first.

Conclusion

- Great paper!
- New insights about self-fulfilling debt crises.
- Striking quantitative results.
- Are self-fulfilling crises quantitatively relevant?
  Yes, according to paper.
- What are their characteristics?
  Rich set of dynamics. Paper could explore that further.

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THANK YOU!