The Nexus of Rollover and Interest Rate Risks in Sovereign Default Models*

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Abstract

This paper proposes a sovereign default model that features interest rate multiplicity driven by rollover risk. The core mechanism exploits a complementarity between traditional notions of slow- and fast-moving crises: the mere possibility of a rollover crisis can lead to high interest rates, which in turn reinforces the rollover risk. The model generates rich simulated debt dynamics with frequent defaults and a volatile bond spread, even without shocks to fundamentals. Calibrated to Spain, it captures its spreads and debt dynamics during the European debt crisis, including the effect of policy announcements and interventions by the European Central Bank.

Keywords: Sovereign default, self-fulfilling crises, multiple equilibria

JEL Classification Numbers: E44, F34

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1 Introduction

Sovereign debt crises are often preceded by rising interest rates and increased debt issuance, which gradually raises the cost of servicing debt and makes financing more difficult. At the heart of the debate lies the question of whether such crises are driven by deteriorating fundamentals (e.g., recessions) or by coordination failures among creditors.

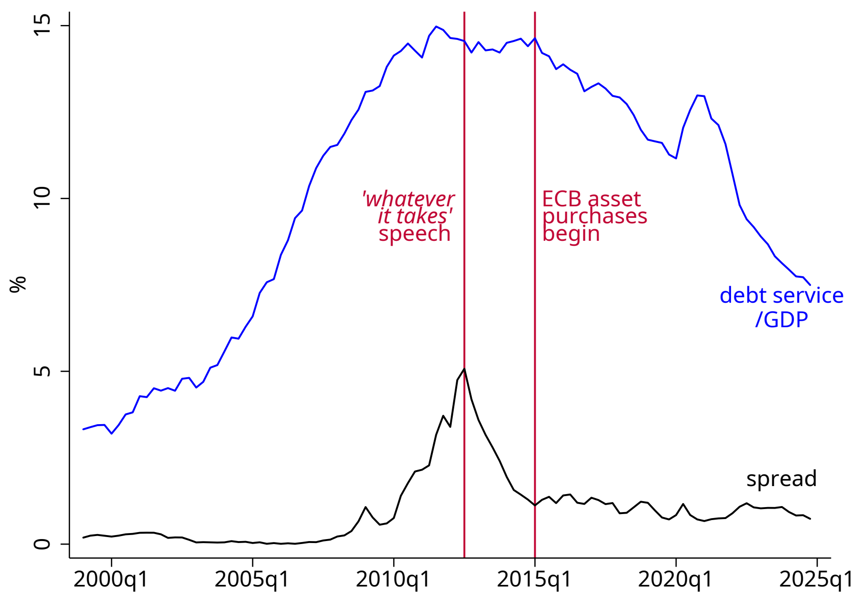


Figure 1: Debt service and interest rate spreads in Spain (2000–2025)

An illustrative example is the European sovereign debt crisis, as shown in Figure 1 for the case of Spain. In his famous speech in July 2012, then-President of the European Central Bank (ECB), Mario Draghi, argued that the rise in sovereign spreads of some Euro Area economies was not justified by economic fundamentals. Instead, he attributed the widening spreads to a crisis of confidence among creditors. This interpretation shaped the ECB's response: first, through Draghi's pledge to do "*whatever it takes*" to preserve the euro; and shortly thereafter, through the announcement of the Outright Monetary Transactions (OMT) program which authorized the ECB to purchase sovereign bonds in the secondary market without ex ante quantitative limits. Sovereign spreads declined significantly following these announcements, without the ECB having to conduct any actual bond purchases under the OMT program.

We develop a quantitative sovereign debt model with self-fulfilling crises in which, even in the absence of fundamental shocks, interest rate spreads and debt issuance dynamics are consistent with Spain's experience during the European sovereign debt crisis, including the effects of policy announcements and interventions by the ECB. The model explores the complementarity between the rollover risk emphasized in Cole and Kehoe (2000) and the interest rate risk highlighted in Calvo (1988). A higher perceived probability of a future rollover crisis raises interest rates today, which in turn increases the likelihood of such a crisis occurring, thereby justifying the higher interest rates through a self-fulfilling mechanism.

The rollover risk in Cole and Kehoe (2000) is motivated by the fact that countries often rely on new borrowing to finance current debt service. If creditors expect an imminent default on the previously issued debt, the price of newly issued bonds is low, which indeed pushes the country to default on the maturing debt because it cannot raise enough revenue. Models of rollover risk have been used to explain fast reversal in capital inflows at the height of a debt crisis, such as December 1994 in Mexico or the summer of 2012 in Europe.[1] A crucial assumption in these models concerns the timing of actions, with the borrower issuing new debt before deciding whether to default on previously issued debt.

In contrast, the interest rate risk in Calvo (1988) is motivated by the fact that higher interest rates can by themselves lead to a buildup in debt service and leave countries at risk of default. Models with this kind of interest rate multiplicity have gained renewed interest recently in an effort to explain the episodes in which bond spreads increase slowly over time, as in Europe starting from 2008.[2] An important assumption in these models concerns the type of actions the borrower can take. We assume the borrower decides on the amount of revenue it needs to raise from bond markets. Therefore, the amount to be repaid in the future is market determined and subject to self-fulfilling crises.[3] High interest rates make default more likely, which in turn justifies the high interest rates.

The European debt crisis, however, was marked by both a gradual buildup in interest rate spreads and, eventually, a sharp reversal in capital inflows. To account for these dynamics, we exploit the complementarity between rollover and interest rate risks, adopting

[1] This source of multiplicity was explored in Cole and Kehoe (1996), Conesa and Kehoe (2017), Bocola and Dovis (2019), Aguiar et al. (2022), and Bianchi and Mondragon (2022).

[2] This source of multiplicity was explored in Aguiar and Amador (2020), Lorenzoni and Werning (2019), and Ayres et al. (2018, 2023).

[3] An alternative is to assume the borrower chooses the amount to be repaid in the future, as in Arellano (2008), which leads to equilibrium uniqueness. See Ayres et al. (2018) and Lorenzoni and Werning (2019).

both the timing assumption in Cole and Kehoe (2000) and the borrower action assumption in Calvo (1988). This yields a simple framework in which a self-fulfilling increase in interest rates endogenously pushes the government into a rollover crisis zone, leaving it vulnerable to further shifts in market sentiment.

We show that while credible policy announcements by a lender of last resort can eliminate high spreads, actual interventions such as third party bailouts are required for the government to exit the rollover risk "crisis zone." This result is consistent with the sequence of interventions that ultimately took place: first, policy announcements by the ECB in the summer of 2012, which successfully reduced spreads but had limited effect on debt-to-GDP ratios; and later, the launch of its bond-buying program in March 2015, after which debt-to-GDP ratios and the corresponding debt service began to decline.

We begin by illustrating our core mechanism in a simple three-period model where the borrower issues Calvo-type debt in the first period and faces a Cole-Kehoe-style rollover risk in the second period. Without any income fluctuations and for a risk-neutral borrower, we show analytically that this setup can produce interest rate multiplicity that is generated by the rollover risk. If creditors expect a rollover crisis in the second period, they will charge a higher interest rate on the debt issued in the first period. But this larger debt payment in the second period may in turn push the borrower to default in case a rollover crisis occurs, hence justifying the higher initial interest rate.

Guided by the results of our stylized three-period model, we extend the analysis to an infinite-horizon setup to test the complementarity between the two sources of multiplicity. We maintain the assumption of no income shocks; hence, the only source of risk comes from two "sunspot" variables that exogenously drive the rollover and interest rate risk sentiments. Calibrated to Spain, the model generates rich debt and bond spread dynamics driven by the interaction between these two forms of multiplicity. Each simulated path eventually enters a "slow-moving debt crisis," in which debt accumulation is fueled by creditor coordination on high interest rates (a bad Calvo sunspot). As debt and interest rates rise, the borrower enters a crisis zone and becomes vulnerable to a rollover crisis (a bad Cole-Kehoe sunspot), which results in a default. While a temporary and unexpected policy announcement by a lender of last resort can bring the spread down to zero, it does not, by itself, reduce debt levels or lead to an exit from the crisis zone. A lasting solution requires actual intervention that delivers sufficient debt relief.

Finally, we augment the model with income shocks estimated using GDP growth data for Spain.[4] The model captures a wide range of combinations of average debt-to-income ratios, as well as the level and volatility of interest rate spreads. In contrast, the pure Cole-Kehoe variant produces a high average spread with no volatility, while the pure Calvo variant generates positive values for both moments, but at magnitudes less than half of those in the baseline. Importantly, the richer dynamics in our model arise primarily from the interaction between the two sources of multiplicity, rather than from simply stacking them together. Thus, our model also demonstrates that a straightforward combination of the two mechanisms of self-fulfilling crises in the literature can create synergy in replicating the empirical moments for debt and spreads that quantitative models typically aim to match.

1.1 Literature review

This paper is closely related to the sovereign default literature with self-fulfilling debt crises. The main two papers that lay foundations for our approach are Calvo (1988) and Cole and Kehoe (2000). Our contribution is to show that a complementarity exists between the two mechanisms, and that it is quantitatively significant. In a closely related paper, Corsetti and Maeng (2024) also study a model with both types of multiplicity to show that the two types of debt crises can arise in a unified framework for different state variables. By contrast, our paper highlights a complementarity between the two modeling assumptions and studies the quantitative implications of their interaction, in particular in a framework with no additional sources of uncertainty.

Our results are also related to recent papers that incorporate equilibrium multiplicity in the spirit of Calvo such as Lorenzoni and Werning (2019), Ayres et al. (2018, 2023), Stangebye (2020) and Bianchi and Bolivar (2025), as well as to those based on the Cole-Kehoe timing assumption, including Conesa and Kehoe (2017), Bocola and Dovis (2019), Aguiar et al. (2022), and Bianchi and Mondragon (2022), among others. Our analysis contributes to this literature along two main dimensions. First, it shows that a simple combination of both sources of multiplicity can generate plausible combinations of debt-to-income ratios, as well as the level and volatility of interest rate spreads. Second, it provides a coherent account of the events surrounding the European debt crisis in the case of Spain. Several of these papers also explore equilibrium multiplicity in the presence of long-term debt. Long-term bonds naturally reduce the scope for rollover crises and allow for alternative

[4] In the Appendix, we also present results for Mexico, a common benchmark in the literature.

forms of multiplicity, as shown in Aguiar and Amador (2020). For this reason, we focus on the case of one-period debt.[5]

Our paper contributes to our understanding of the forces at play during the European debt crisis. Using models based on fundamentals, Paluszynski (2023) explains the gradual development of that episode, while Paluszynski and Stefanidis (2023) show that frictions in spending adjustment may explain why governments simultaneously increased their external debt. The present paper achieves similar objectives in a model where the debt crisis is self-fulfilling and consistent with the observed effects of the ECB's policy announcements.

2 Multiplicity in a three-period model

This section presents a simple three-period environment to illustrate our core mechanism. For simplicity, we present the derivation of our main result for a risk-neutral borrower. In Appendix A, and in the subsequent infinite-horizon model, we assume a risk-averse borrower.

The borrower receives *deterministic* endowment y in all three periods ($t = 0, 1, 2$). It has zero initial debt and can issue one-period non-contingent bonds to competitive risk-neutral lenders. The borrower is not committed to repay the debt. In the case of default, he is permanently excluded from international financial markets and restricted to consume $y^d < y$. The risk-free gross interest rate is denoted by R^*. To induce borrowing, we assume the borrower has a lower discount factor than the lenders, denoted by β.

As in Cole and Kehoe (2000), we assume the borrower chooses whether or not to default on the previously issued debt *after* the new debt issuance takes place.[6] In this setting, lenders may not roll over the debt if the lack of new borrowing pushes the borrower to default on the old debt, which characterizes the rollover risk. As in Eaton and Gersovitz (1981), we assume that when the bond auction takes place, the borrower moves first by committing to the amount of resources he wishes to raise in the current period, denoted by b. Lenders move next and set the gross interest rate R. These assumptions generate

[5]While our main results extend to a version of the model with a moderate profile of long-term debt, a comprehensive analysis of that variant is beyond the scope of this paper.

[6]We do not allow for randomization over the default decision.

the interest rate multiplicity, as in Calvo (1988).[7] For a given b, a higher R increases the probability of default because it increases the debt service. In turn, a higher probability of default implies a higher R, as lenders must have an expected return equal to R^* in equilibrium.

We present and solve the problem backward. In period $t = 2$, the only choice for the borrower is whether to repay the debt issued in the previous period, $R_2 b_2$, or to default. The borrower defaults if $y^d > y - R_2 b_2$ and repays otherwise.

It follows that in period $t = 1$, if the lenders roll over the debt, the interest rate is uniquely determined. Let us define the threshold $\widetilde{B}_2 \equiv \left(y - y^d\right)/R^*$. For $b_2 \leq \widetilde{B}_2$, there is no default and R_2 must be equal to R^*. For $b_2 > \widetilde{B}_2$, the borrower defaults for sure, so $\widetilde{B}_2$ becomes a borrowing limit.

If lenders do not roll over the debt in $t = 1$, the borrower defaults if

$$v_1^d \equiv (1+\beta)y^d > y - R_1 b_1 + \beta y,$$

where $R_1 b_1$ is the debt service on the debt issued in $t = 0$.[8] A rollover crisis may happen only if it pushes the country to default, so the borrower is subject to rollover risk only if

$$R_1 b_1 > (1+\beta)(y - y^d). \tag{1}$$

Note that a rollover crisis is equivalent to setting the borrowing limit to zero, a convention we will adopt to simplify the exposition. We can express the problem in period $t = 1$ as

$$v_1\left(R_1 b_1, s_{ck}\right) = \max\{v_1^{nd}\left(R_1 b_1, s_{ck}\right), v_1^d\},$$

where

$$v_1^{nd}\left(R_1 b_1, s_{ck}\right) = \max_{b_2 \leq \overline{B}_2(R_1 b_1, s_{ck})} y - R_1 b_1 + b_2 + \beta\left(y - R^* b_2\right).$$

The sunspot variable $s_{ck} \in \{0, 1\}$ commands the Cole-Kehoe type of market sentiment. If $s_{ck} = 0$ and condition (1) holds, a rollover crisis happens and the borrowing limit $\overline{B}_2\left(R_1 b_1, s_{ck}\right)$ equals zero. Otherwise, $\overline{B}_2\left(R_1 b_1, s_{ck}\right) = \widetilde{B}_2$.[9] In addition, note that the

[7]See Ayres et al. (2023).

[8]As in Aguiar et al. (2016), we assume the borrower does not keep the proceeds from the new bond auction in case it defaults on the old debt.

[9]Note that the optimal strategy for the borrower in this simple case is to set $b_2 = \overline{B}_2\left(R_1 b_1, s_{ck}\right)$.

condition for the rollover risk in (1) depends on R_1, which gives rise to interest rate multiplicity. For a given b_1, higher R_1 makes a rollover crisis more likely. In turn, the higher probability of a rollover crisis implies a higher R_1.

We now turn to the borrower's problem in $t = 0$:

$$v_0(s_c) = \max_{b_1 \leq \bar{b}_1} y + b_1 + \beta \sum_{s_{ck} \in \{0,1\}} \pi(s_{ck}) \, v_1\Big(R_1(b_1, s_c)b_1, s_{ck}\Big),$$

where $\pi(s_{ck})$ denotes the probability distribution over the values that s_{ck} may take in $t = 1$. We let p denote the probability of the bad sunspot, $\pi(0) = p$. The state variable $s_c \in \{0, 1\}$ denotes the Calvo-type sunspot. In case there are multiple interest rates for a given b_1 such that lenders receive an expected return equal to R^*, we use the sunspot variable s_c as a device to select the interest rate. As in Ayres et al. (2023), we will focus on two extreme cases. In the bad sunspot state, $s_c = 0$, R_1 takes the highest possible value. In the good sunspot state, $s_c = 1$, R_1 takes the lowest possible value. Proposition 1 characterizes all pairs (b_1, R_1) such that lenders receive return R^* in expectation.

Proposition 1 *The pairs (b_1, R_1) in which lenders receive an expected return equal to R^* given the borrower's optimal borrowing and default strategies are:*

(i) $b_1 \leq \frac{(1+\beta)(y-y^d)}{R^*} \equiv \underline{\underline{B}}_1$ *and* $R_1 = R^*$.

(ii) $\underline{B}_1 \equiv \frac{(1+\beta)(y-y^d)(1-p)}{R^*} \leq b_1 \leq (1-p)(y-y^d)\left(\frac{1}{R^*} + \frac{1}{(R^*)^2}\right) \equiv \overline{B}_1$ *and* $R_1 = \frac{R^*}{1-p}$.

Proof Recall that the borrower will default in period $t = 1$ if $R_1 b_1 > (1+\beta)(y - y^d)$. Because there are two possible values for the interest rate, R^* and $R^*/(1-p)$, we have two debt thresholds that limit the repayment decision in the case of low and high interest rates:

$$b_1 \leq \frac{(1+\beta)(y-y^d)}{R^*} \equiv \underline{\underline{B}}_1$$
$$b_1 \leq \frac{(1+\beta)(y-y^d)(1-p)}{R^*} \equiv \underline{B}_1.$$

Finally, we need to find a debt threshold that makes the borrower indifferent between repaying and defaulting when markets are open in period $t = 1$. The condition is

$$v_1^d = (1+\beta)y^d = y - \frac{R^*}{1-p} b_1 + b_2 + \beta \max\{y - R_2 b_2, y^d\}.$$

Under risk neutrality, assuming the borrower is impatient enough, the optimal borrowing in that period is $b_2^* = \frac{y-y^d}{R^*}$. Plugging this value into the indifference condition yields the following upper debt threshold:

$$b_1 = (1-p)(y-y^d)\left(\frac{1+R^*}{R^{*2}}\right) \equiv \overline{B}_1.$$

■

For any debt level smaller than $\underline{B}_1$, the borrower repays even if lenders do not roll over the debt. Hence, the interest rate is unique and equal to the risk-free rate. For a level of borrowing between $\underline{B}_1$ and $\underline{\underline{B}}_1$, however, multiple interest rates arise. It is noteworthy that, in this case, it is the rollover risk in period $t = 1$, rather than the income shock, that drives the multiplicity of the interest rate. In other words, for a given $b_1 \in \left[\underline{B}_1, \underline{\underline{B}}_1\right]$, a high or low interest rate R_1 determines whether or not the borrower finds itself in a Cole-Kehoe type of crisis zone. Finally, if the debt level is sufficiently high, above $\underline{\underline{B}}_1$, then the interest rate is again unique and equal to the high rate $\frac{R^*}{1-p}$. At such debt levels, the borrower is in the crisis zone unconditionally and defaults in the event of a bad s_{ck} sunspot even if interest rates were set to R^*.

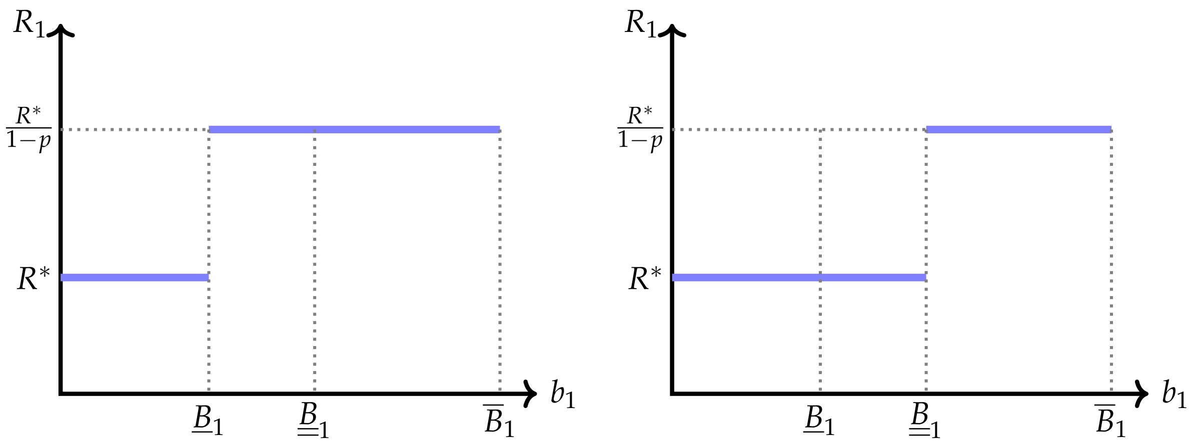
The borrower observes s_c before the bond auction and internalizes how the interest rate will vary with respect to the amount of debt it chooses to issue. Therefore, when choosing how much to borrow, he considers an interest rate schedule $R_1(b_1, s_c)$ as a mapping from debt levels into unique interest rate values. Figure 2 presents a stylized illustration of the two interest rate schedules the borrower may face. Panel 2(a) features the case of the higher rate within the multiplicity interval, $R_1(b_1, 0)$, while Panel 2(b) features the case of the lower rate, $R_1(b_1, 1)$.

3 Infinite-horizon model

In this section, we develop an infinite-horizon model to study the interaction between interest rate multiplicity and rollover risk.

3.1 Economic environment

Consider a small open economy with a benevolent sovereign that borrows internationally from competitive lenders and receives a stochastic endowment. Time is discrete and indexed by $t = 0, 1, 2, ...$ Markets are incomplete, and the only asset available for trad-



(a) High interest rate schedule $R_1(b_1, 0)$ (b) Low interest rate schedule $R_1(b_1, 1)$

Figure 2: Stylized illustration of the interest rate schedules

ing is the one-period non-contingent bond. The risk-free gross interest rate is R^*. The representative household has preferences given by the expected utility of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where we assume the function $u(\cdot)$ is strictly increasing, concave, and twice continuously differentiable. The discount factor is given by $\beta \in (0, 1)$.

Income process The economy's income is affected by stochastic endowment growth realizations and evolves according to

$$Y_t = g_t Y_{t-1}, \tag{3}$$

where g_t denotes the growth shock. The growth rate can take two values, g_L and g_H, with $g_H > g_L$. It follows a Markov process with the transition probability matrix given by

$$\Pi = \begin{bmatrix} \pi_L & 1 - \pi_L \\ 1 - \pi_H & \pi_H \end{bmatrix}, \tag{4}$$

where $Pr(g_{t+1} = g_L | g_t = g_L) = \pi_L$ and $Pr(g_{t+1} = g_H | g_t = g_H) = \pi_H$. We model the income shock process as growth regimes following Ayres et al. (2018), who show that a bimodal income process generates Calvo-style interest rate multiplicity.[10] That said, our

[10] It is also possible for the model to feature a transitory shock, but its variance cannot be too large.

main quantitative result in Section 3.3 is obtained with a variant of the model with no income shocks whatsoever.

Timing The timing assumptions are the same as in Section 2. The borrower chooses whether or not to default on the debt from the previous period *after* the new debt issuance (Cole and Kehoe, 2000). Similar to Calvo (1988), when the bond auction takes place, the borrower moves first by committing to the amount of revenue it wishes to raise from bond markets in the current period, b. Lenders move next and set the gross interest rate R. Shocks are observed at the beginning of the period.

States The state variables are $\{A, Y_{-1}, g, \mathbf{s}\}$. $A = RB$ denotes the total debt service to be paid in the current period, $Y = gY_{-1}$ is the current income, and $\mathbf{s} = \{s_c, s_{ck}\}$ is a vector of sunspot realizations corresponding to the interest rate multiplicity and rollover risk, respectively. To make the problem stationary, all value functions are ultimately normalized by the previous income level Y_{-1}.

Recursive problem The value function of the government involves a choice of whether or not to default:

$$V(A, Y_{-1}, g, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ (1-d) V^{nd}(A, Y_{-1}, g, \mathbf{s}) + d V^{d}(Y_{-1}, g, \mathbf{s}) \right\}.$$

The value associated with repayment is

$$V^{nd}(A, Y_{-1}, g, \mathbf{s}) = \max_{B' \leq \overline{B}(A, Y_{-1}, g, \mathbf{s})} \left\{ u(C) + \beta \sum_{g'} \sum_{\mathbf{s}'} \Pi(g'|g) p(\mathbf{s}'|\mathbf{s}) V\left(B' R(B', Y_{-1}, g, \mathbf{s}), Y, g', \mathbf{s}'\right) \right\} \tag{5}$$

subject to

$$C = Y - A + B'.$$

The value associated with default is

$$V^{d}(Y_{-1}, g, \mathbf{s}) = u\Big(Y(1-\phi)\Big) + \beta \sum_{g'} \sum_{\mathbf{s}'} \Pi(g'|g) p(\mathbf{s}'|\mathbf{s}) \left\{ \theta V\left(0, Y, g', \mathbf{s}'\right) + (1-\theta) V^{d}\left(Y, g', \mathbf{s}'\right) \right\},$$

where ϕ represents the fraction of income lost upon default. We assume that, following a default, the borrower has probability θ of being readmitted to capital markets in each period and the recovery rate of defaulted debt is zero.

As in Section 2, the borrowing limit $\overline{B}\left(A, Y_{-1}, g, \mathbf{s}\right)$ equals zero whenever $s_{ck} = 0$, and the lack of new borrowing pushes the country to default. That happens when the following condition is satisfied:

$$u(Y - A) + \beta \sum_{g'} \sum_{\mathbf{s}'} \Pi(g'|g) p(\mathbf{s}'|\mathbf{s}) V\left(0, Y, g', \mathbf{s}'\right) \leq V^d(Y_{-1}, g, \mathbf{s}).$$

Definition 1 formally defines an equilibrium in this economy.

Definition 1 *A Markov perfect equilibrium for this economy consists of the government value functions $V(A, Y_{-1}, g, \mathbf{s})$, $V^{nd}(A, Y_{-1}, g, \mathbf{s})$, $V^d(Y_{-1}, g, \mathbf{s})$; policy functions $B'(A, Y_{-1}, g, \mathbf{s})$ and $d(A, Y_{-1}, g, \mathbf{s})$; the interest rate schedule $R(B', Y_{-1}, g, \mathbf{s})$ and the function for the borrowing limit $\overline{B}\left(A, Y_{-1}, g, \mathbf{s}\right)$ such that:*

1. *Policy function $d(A, Y_{-1}, g, \mathbf{s})$ solves the government's default-repayment problem.*
2. *Policy functions $B'(A, Y_{-1}, g, \mathbf{s})$ solve the government's consumption-saving problem.*
3. *Interest rate schedules $R(B', Y_{-1}, g, \mathbf{s})$ and borrowing limit functions $\overline{B}\left(A, Y_{-1}, g, \mathbf{s}\right)$ are such that international lenders receive an expected return equal to R^*.*

3.2 Quantification of the model

In this section, we parameterize our model in order to evaluate its quantitative performance.[11] We calibrate the variant of our model with no growth shocks to match the experience of Spain in years 2010-2015. Specifically, the risk-free rate r is set to 0.01, the risk aversion γ is 2, and the probability of re-entry following a default θ is 0.125, all standard values (Aguiar et al., 2022). Then, the discount factor β and the income loss in default ϕ are picked so that debt crises in the model (the mechanics of which we describe in next section) replicate the mean amount of debt service to GDP of Spain in years 2010-2015 of 14.5% and its peak bond spread of 5%, as shown in Figure 1. The resulting values for β and ϕ are 0.813 and 0.022, respectively.[12]

[11] We solve for a detrended version of the model in Section 3.1, following the approach of Aguiar and Gopinath (2006) and Ayres et al. (2023).

[12] In Appendix D we conduct an alternative calibration for Mexico.

3.3 No growth shocks

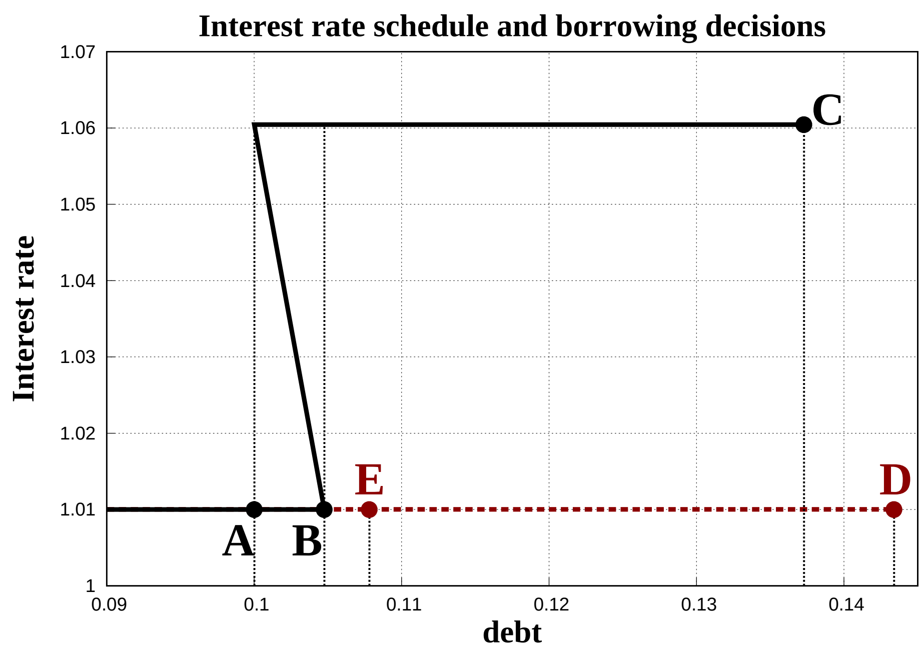
Simulated behavior As a first step, we evaluate the model with no growth regimes (thus, no fundamental shocks whatsoever). Income is deterministic and equal to 1 in every period. Hence, in this variant of the model, rollover risk is the sole driver of defaults and a potential interest rate multiplicity. We start with 0.1 as the initial probability of the bad Calvo sunspot realization ($s_c = 0$), and we vary the probability of the bad Cole-Kehoe sunspot to illustrate how the model works. Columns (1)–(3) in Table 1 present the statistics from a simulated ergodic distribution for three different probabilities of a bad Cole-Kehoe sunspot ($s_{ck} = 0$). As is evident, the model features starkly different types of behavior for seemingly similar values of this parameter. When the probability is about 4.7% or lower, the agent borrows on the higher interest rate schedule and defaults every time a rollover crisis occurs. As a result, the average spread is roughly equal to the probability of a bad Cole-Kehoe sunspot, while the variance of the spread is zero. On the other hand, for a probability of about 4.9% or higher, the agent borrows on the lower interest rate schedule and reduces debt every time the Calvo sunspot switches to bad in order to avoid the region of multiplicity. As a result, no defaults occur on the equilibrium path and the bond spread is exactly zero. In between the two extremes, there is an interval of the bad Cole-Kehoe sunspot probabilities centered around the value of 4.8% where interesting action occurs. In this case, the agent initially borrows on the lower interest

Table 1: Simulation results in the quantitative model calibrated to Spain

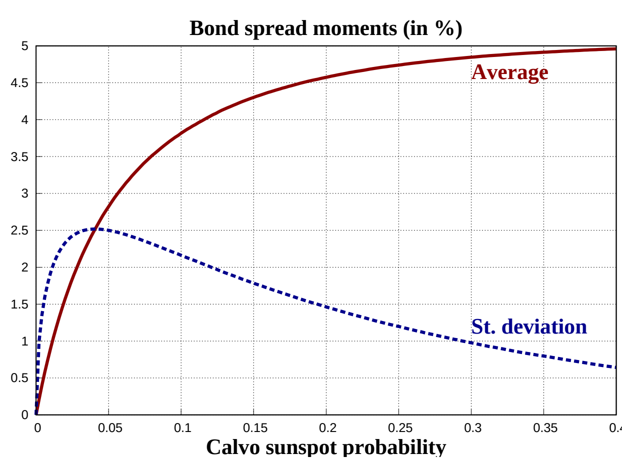
Statistic	**Model without growth regimes**			**Model with growth regimes**		**Data**
	(1)	(2)	(3)	(4)	(5)	(6)
$P(s_c = 0)$	0.10	0.10	0.10	0.24	0.00	
$P(s_{ck} = 0)$	0.047	0.048	0.049	0.02	0.00	
avg(A/Y)	14.6%	13.4%	10.5%	10.3%	24.0%	10.3%
avg($spreads$)	5.0%	3.8%	0.0%	1.0%	0.0%	1.0%
std($spreads$)	0.0%	2.2%	0.0%	0.9%	0.0%	1.0%
corr($spreads, TB$)	0	0.36	0	-0.09	0.0	0.51
corr(TB, Y)	-	-	-	-0.18	-0.52	-0.22
corr($spreads, Y$)	-	-	-	0.16	0.0	-0.41

Note: $P(s_{ck} = 0)$ and $P(s_c = 0)$ denote the probabilities of a bad Cole-Kehoe-type and Calvo-type sunspot realization, respectively. Y = output, A = debt service, TB = trade balance-to-output ratio. All statistics are computed over non-default periods.

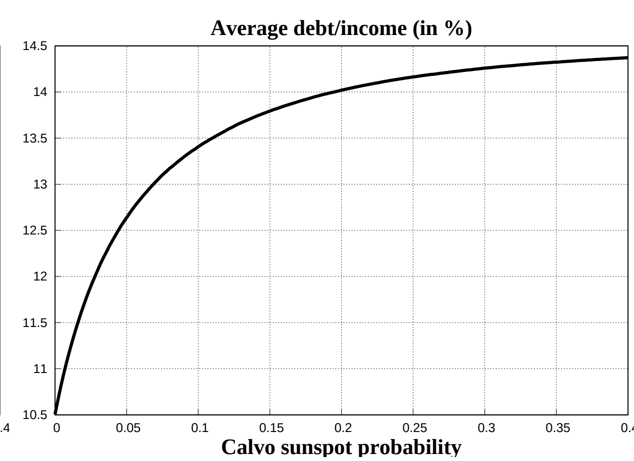
rate schedule but then increases the debt and jumps to the higher rate when the Calvo sunspot switches to bad (a "slow-moving debt crisis"). The borrower remains there until a Cole-Kehoe-type rollover crisis forces him into default. Consequently, the simulated bond spread exhibits high average and volatility. It should be emphasized that while the interval of sunspot probabilities for which the interesting behavior occurs is narrow, it is so because the model does not feature any other sources of uncertainty. Section 3.4 shows that this interval widens considerably when income shocks are introduced.



(a) Interest rate schedules and policy functions



(b) Simulated bond spread moments



(c) Simulated average debt ratio

Figure 3: Simulated moments as function of the Calvo sunspot probability

Model mechanics Figure 3(a) plots the interest rate schedule in the intermediate case of $P(s_{ck} = 0) = 0.048$. The clear multiplicity interval confirms our analytical result from Section 2, which shows that the Calvo action space can combine with rollover crises to generate overlapping interest rate schedules *with no income shocks*. We also use the graph to describe the dynamics of the borrower's decisions in this model.[13] As the agent accumulates debt starting from zero, he moves along the risk-free interest rate toward the points labeled "A" and "B." The former is chosen if the Calvo sunspot realization is initially bad, whereas the latter is eventually selected when the realization switches to good. Once the borrower lands at point B, he will not retreat to point A upon another bad Calvo sunspot, but instead will borrow all the way to point C and incur an interest rate of 6%. With no additional friction or shocks in the model, the agent stays at point C until a Cole-Kehoe rollover crisis occurs, in which case he defaults. This behavior contrasts with that reported in Ayres et al. (2023) for the case of Spain, where the government responds with endogenous austerity (associated with choosing point A), thereby preventing the higher interest rates driven by pessimistic expectations from materializing in equilibrium.

Policy interventions Figure 3(a) also features points D and E which we refer to as policy interventions. First, suppose that while the borrower is in point C, a lender of last resort unexpectedly steps in and eliminates the possibility of a rollover crisis for one period.[14] In such case, the interest rate spread drops to zero in that given period while the borrower increases the face value of the debt slightly to keep the debt obligation constant (point D). Importantly, this type of intervention offers no possibility of reducing the debt and escaping the Cole-Kehoe crisis zone. If the lender of last resort withdraws its pledge, the borrower will revert back to point C. The only way for the government to exit the crisis zone is through debt relief. If the Calvo sunspot realization is favorable, the required transfer does not need to cover the entire gap between points C/D and B. Instead, it is sufficient to reduce indebtedness to an endogenous point E, at which the government will optimally choose to return to B. If the Calvo sunspot realization is unfavorable, however, the debt relief must cover the full distance between points C/D and A.

The two types of intervention we consider resonate with the aftermath of the Spanish debt crisis in the summer of 2012. As Figure 1 shows, the European Central Bank succeeded in bringing down the Spanish bond spread with a combination of its "Whatever

[13] Appendix B provides a more detailed analysis of this behavior by examining the full set of policy functions corresponding to columns (1)–(3) in Table 1.

[14] In the case of a permanent elimination of a rollover crisis, the model becomes degenerate as the impatient government borrows up to the fundamental debt limit and never defaults.

it takes" pledge and the announcement of the OMT bond purchase program. However, it failed to induce the government to reduce the debt burden. The gradual debt reduction only materialized starting from 2015 when the ECB's asset purchase program was launched, amounting to a de facto debt relief. The exact value of this implicit transfer is difficult to assess because we do not know the counterfactual spread on Spanish bonds in the absence of this intervention. However, by simply examining the magnitudes and considering various assumptions, in Appendix C we show that the total value of such a transfer may have varied between 0.5% up to 10% of Spain's GDP in 2015, with the intermediate parameterization showing a debt relief of 3.6%. This is consistent with the difference between points D and E in Figure 3(a) of around 3.5%, representing the amount of debt relief needed to exit the Cole-Kehoe crisis zone and return to point B.

The temporary policies discussed thus far take effect after the country has already issued debt at high interest rates. An alternative set of policies, however, can be implemented ex-ante to prevent the country from reaching such elevated rates in the first place. That is, the government could be prevented from ever moving from point B to point C in Figure 3(a). Specifically, following a bad Calvo sunspot realization, a transfer that reduces debt service back to point A would be sufficient. At that point, the government would optimally choose not to accumulate additional debt. The fiscal cost of this ex-ante policy is substantially lower than that of rescuing the government once it enters the rollover crisis zone, approximately 0.5% of GDP, compared to around 3.5% in the latter case.

Comparative statics An interesting aspect of our model is that the interval of the bad Cole-Kehoe sunspot probabilities that generate these dynamics is the same for any bad Calvo sunspot probability parameter that we choose. However, the implications for the simulated moments are quite different as we vary the likelihood of a Calvo-style buildup. Panels 3(b) and 3(c) of Figure 3 explore these comparative statics by plotting the key moments of the bond spread and debt for a range of values that this parameter can take. We find that the average spread and average debt ratio are both monotonically increasing in the probability of the bad Calvo sunspot. The intuition is simple: as switching to the higher interest rate schedule becomes more likely, the borrower spends less time at point B of Figure 3(a), characterized by lower debt and zero spread, and more time at point C, with high debt and positive spread. On the other hand, the measured volatility of the spread is *non-monotonic*, initially rising sharply from zero and then falling back gradually. Intuitively, if a Calvo-style crisis is unlikely, or if it happens often, the borrower will end up spending a disproportionate amount of time on the lower or upper interest rate schedule, respectively. Hence, there exists an intermediate value for the bad Calvo sunspot probability that balances the average time spent on the two parts of the schedule and maximizes the overall bond spread volatility. For the present parameterization, we find that the standard deviation of the bond spread peaks at around 2.5% for the bad Calvo sunspot probability of around 5%.

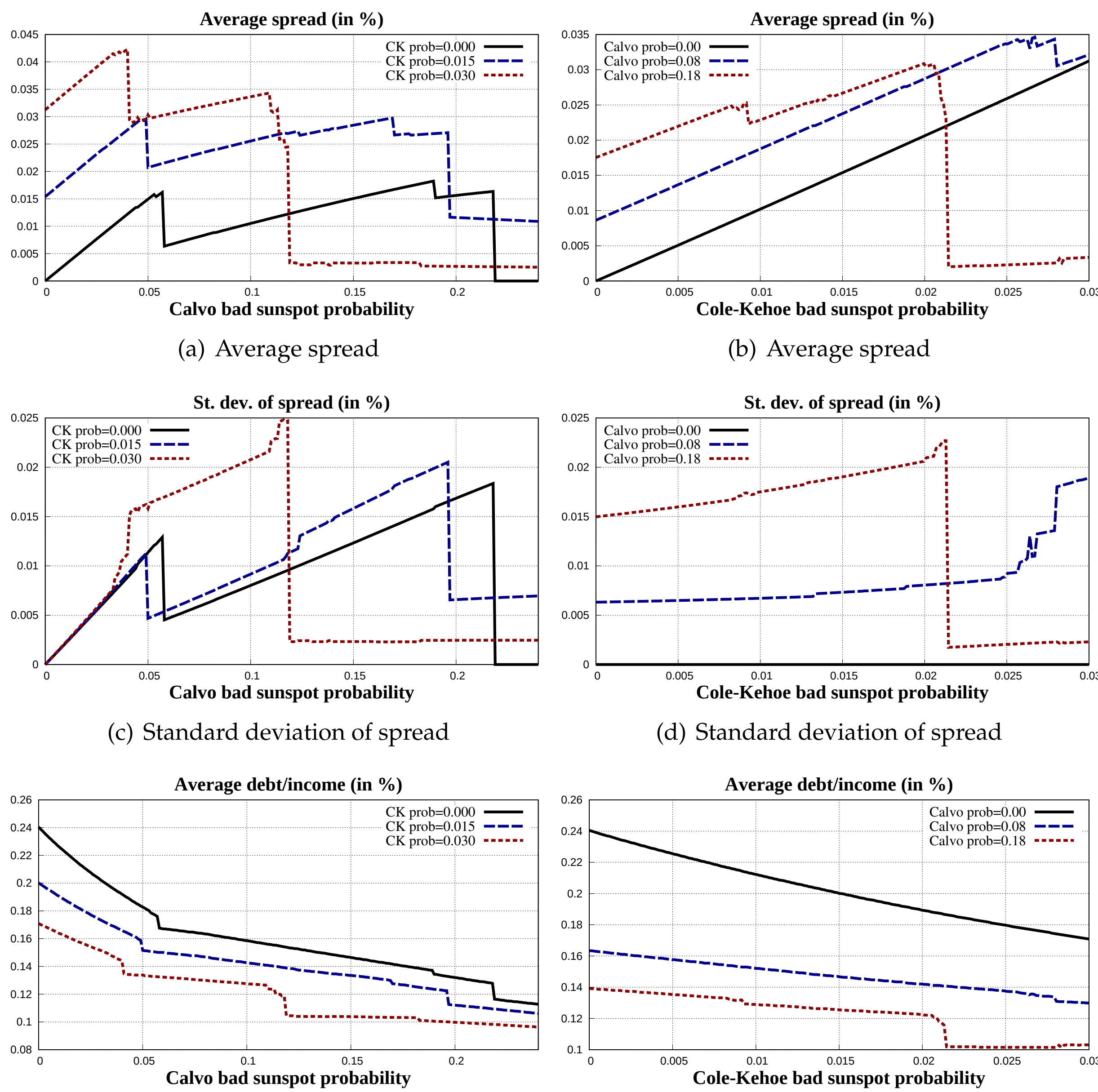
3.4 Quantitative results with growth regimes

We now evaluate the variant of our model with income shocks and we use it to conduct comparative statics analysis. We borrow the parameters of a Markov-switching process for growth shocks in Spanish real GDP from Ayres et al. (2023). Their estimates for the high and low regime growth rates are $g_h = 1.034$ and $g_l = 0.982$, while the persistence of high and low regimes is $\pi_H = 0.84$ and $\pi_L = 0.63$, respectively. Given the estimated income process, and keeping the structural parameter values β and ϕ from Section 3.3 unchanged, we then pick the combination of bad Calvo and the Cole-Kehoe sunspot probabilities to match the average debt and average bond spread for the entire data episode.

Column (4) in Table 1 presents the simulated results. We find that the combination of the Calvo bad sunspot probability of 21% and the Cole-Kehoe bad sunspot probability of 2% yield an average levels of debt and spread close to the empirical targets for the sample of 1999-2024, reported in Column (6) of Table 1. Importantly, it also allows the model to nearly match the volatility of the spread of around 1%. In Appendix D, we do the same exercise for the case of Mexico, an emerging market economy, which features a higher debt-to-income ratio and a considerably higher average and standard deviation of the spread. Our model is able to match these moments equally well which entails a slightly lower probability of a Calvo-style buildup, and a higher probability of a rollover crisis.

Column (5) in Table 1 further shows that sunspots play an essential role in the quantitative results, as both the average spreads and the volatility of spreads drop to zero when the probability of bad sunspot realizations is eliminated.

Comparative statics We now turn to the comparative statics analysis of our model (using the calibration for Spain) with respect to the bad sunspot probabilities. Panels 4(a), 4(c), and 4(e) of Figure 4 plot the three moments of interest while varying the bad Calvo sunspot probability up to 0.25, for three different values of the bad Cole-Kehoe sunspot probability, while panels 4(b), 4(d), and 4(f) plot it the other way round. In particular, the solid black line shows the moments generated by either the pure Calvo or the pure Cole-Kehoe model. There are two main observations to take away from these graphs. First, by varying both probabilities together, our model is capable of generating a much wider range of combinations of the debt and spread moments than a model with a single source of multiplicity. In particular, such models display a pervasive knife-edge behavior with the borrower either choosing to be exposed with high debt and high spread, or avoiding



(a) Average spread

(b) Average spread

(c) Standard deviation of spread

(d) Standard deviation of spread

(e) Average debt

(f) Average debt

Figure 4: Comparative statics in the baseline calibration for Spain

such exposure altogether and exiting the crisis zone. While our model is also characterized by such threshold behavior, it features many more combinations of expansions and reductions in borrowing, and as a result it is capable of matching the combination of empirical moments for either Spain, as Table 1 shows, or Mexico, as Table 3 in Appendix D shows.

Second, the two sources of multiplicity interact in driving the variation in the bond spread. To see this, notice first that for the Calvo sunspot probabilities up to around 0.05, the average spread increases roughly by the probability of a rollover crisis, while the standard deviation of the spread is the same (as the pure Cole-Kehoe model does not generate any volatility in the bond spread). This implies that the two sources of multiplicity in this region are independent of each other and their effects are approximately additive. Beyond this threshold, however, an interesting interaction occurs. As we add rollover crises, the average spread rises by less than the probability of a bad Cole-Kehoe sunspot, while the standard deviation of the spread rises by more. At the peak, the standard deviation reaches 2.5% in our model, compared to 1.8% in the pure Calvo and zero volatility in the pure Cole-Kehoe variants.

4 Conclusion

This paper contributes to the literature on self-fulfilling debt crises by introducing a simple model with interest rate multiplicity generated by belief-driven runs on government debt. In turn, such runs are justified by a realization of high interest rates that by itself results from pessimistic beliefs. The most interesting feature of the model is that it generates rich dynamics of sovereign debt and the interest rate spread by interacting the notions of slow- and fast-moving debt crises *without any underlying shocks to fundamentals*. Through a combination of simplicity and quantitative rigor, the model allows us to simultaneously think about the slow- and fast-moving stages of the European debt crisis of 2008-2012, as illustrated by our case study of Spain.

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Appendices (for online publication)

A Three-period model with risk-averse borrower

In this section we derive the borrowing thresholds for the case of a risk-averse borrower. We assume a CRRA utility function of the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and we analyze the problem backward. Similar to the case of risk neutrality, in period $t = 2$ the agent repays if $y - b_2 R^* \geq y^d$. In period $t = 1$, if markets do not roll over the debt, the borrower will default if

$$v_1^d = (1+\beta)u(y^d) > v_1(R_1 b_1, s_1 = 1) = u(y - R_1 b_1) + \beta u(y)$$

If $\gamma > 1$, this condition boils down to

$$R_1 b_1 > y - \left((1+\beta)(y^d)^{1-\gamma}\right)^{\frac{1}{1-\gamma}},$$

which shows that the default decision depends on the level of interest rate. Consequently, we have the two debt thresholds that limit the repayment decision for the case of low and high interest rates:

$$b_1 \leq \frac{1}{R^*}\left[y - \left((1+\beta)(y^d)^{1-\gamma} - \beta y^{1-\gamma}\right)^{\frac{1}{1-\gamma}}\right] \equiv \underline{\underline{B}}_1$$
$$b_1 \leq \frac{1-p}{R^*}\left[y - \left((1+\beta)(y^d)^{1-\gamma} - \beta y^{1-\gamma}\right)^{\frac{1}{1-\gamma}}\right] \equiv \underline{B}_1.$$

Next, to find the debt threshold that makes the borrower indifferent between repaying and defaulting when markets are open in $t = 1$, we need to find optimal borrowing b_2. Under risk aversion, this entails solving the problem

$$v_1(R_1 b_1, s_1 = 0) = \max_{b_2} \; u(y - R_1 b_1 + b_2) + \beta u(y - R^* b_2).$$

The interior solution to this problem is $b_2^* = \frac{(\beta R^*)^{-1/\gamma} y - (y - R_1 b_1)}{1 + (\beta R^*)^{-1/\gamma} R^*}$, while a corner implies $b_2^* = \frac{y - y^d}{R^*}$. To find threshold $\overline{B}_1$, we need to plug this into the indifference condition in period $t = 1$,

$$v_1^d = (1+\beta)u(y^d) = u(y - \frac{R^*}{1-p} b_1 + b_2^*) + \beta u(y - R^* b_2^*),$$

and solve for b_1. Under risk aversion, this solution cannot be obtained analytically. The results in Section A.1 present our numerical solution to this problem.

A.1 Numerical example

In this subsection, we provide a simple numerical example to show that the interest rate multiplicity characterized so far is realistic. We also extend the analysis to the case of a risk-averse borrower and show that the result becomes even stronger. The exact derivations for this case are presented in Appendix A.

Consider the case of a risk-averse borrower with a CRRA utility function of the form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We assume the following, fairly realistic parameterization: $\beta = 0.7$, $\gamma = 3$, $y = 1$, $y^d = 0.95$, $p = 0.8$, $R = 1.03$. Figure 5 presents the interest rate schedules, as well as the optimal debt policy for the risk-averse borrower. The solid blue line depicts the lower (risk-free) interest rate, while the dashed red and dotted blue lines represent the upper (risky) interest rate for the case of a risk-averse and risk-neutral borrower, respectively. It is immediate to notice that including risk aversion causes the interest rate multiplicity to almost double in size. The presence of this multiplicity also has real consequences for the borrower's actions. When the Calvo sunspot is bad, the government must reduce its debt by around 20%, compared to the case of a good sunspot, to avoid the higher interest rate.

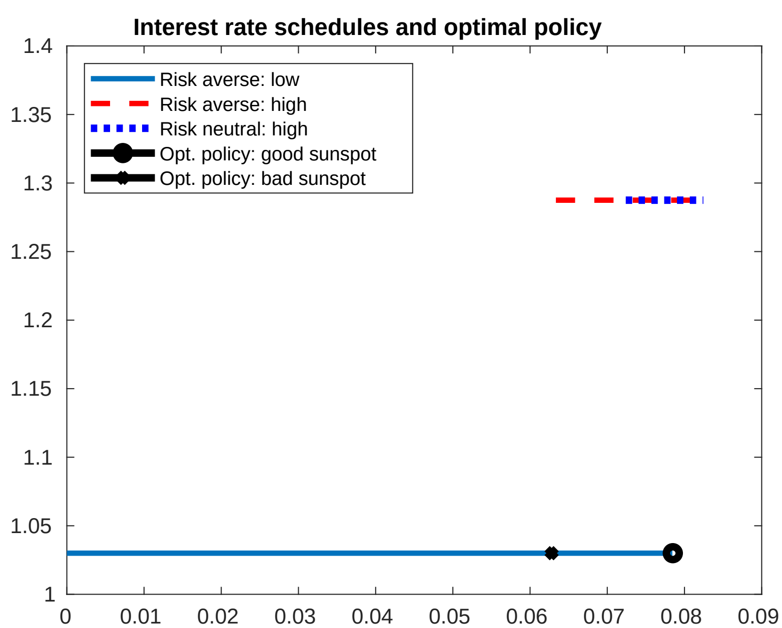


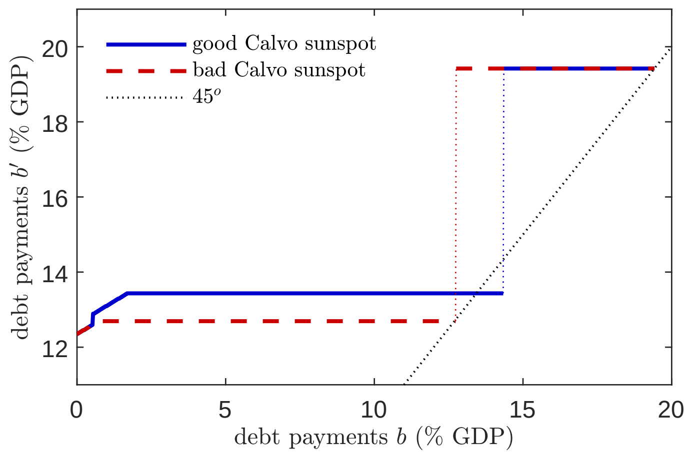
Figure 5: Interest rate schedules and optimal policy

B Further illustrations for the infinite-horizon model

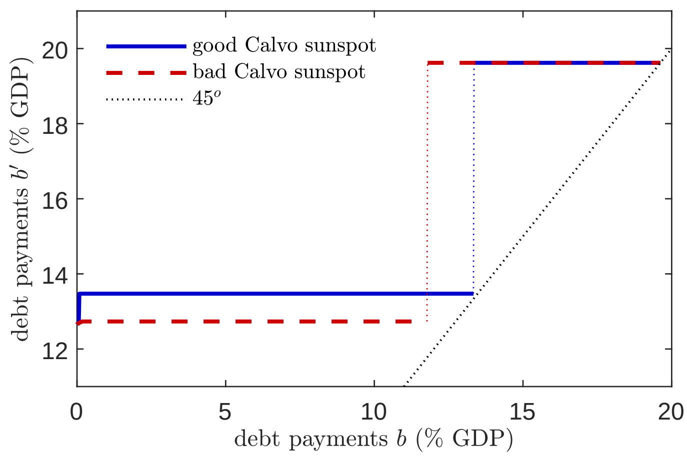
In this section, we provide further illustration of the government's borrowing choices in the model with no income shocks, corresponding to the three types of behavior presented in columns (1)–(3) in Table 1. Panel 6(a) of Figure 6 shows the optimal next period debt choice (b') as a function of today's debt (b), for the two possible realizations of the Calvo sunspot. As the government accumulates debt from zero, it will ultimately stop at the 45 degree line, with the exact amount depending on the current realization of the Calvo sunspot. If the sunspot is bad, then the government will stop just outside of the interval of interest rate multiplicity (corresponding to point A in Figure 3(a)). If the sunspot is good, the government will borrow more (progressing to point B), at which point another bad sunspot realization will induce it to jump to a debt level of about 19% with no possibility of retreat.

Panel 6(b) of Figure 6 illustrates the case corresponding to column (1) in Table 1, which we refer to as “exposed." As the government accumulates debt from zero, none of the lower segments of the policy functions actually cross the 45 degree line. This means that regardless of the realization of the Calvo sunspot, the government will end up jumping to a level of debt of around 18%, which is the region of the Cole-Kehoe-type crisis zone. As a result, while the average bond spread is high, it does not exhibit any volatility over time.

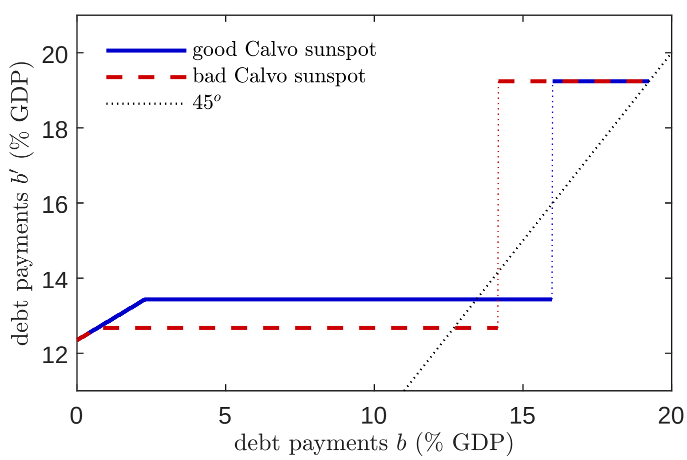
Finally, Panel 6(c) of Figure 6 illustrates the case corresponding to column (3) in Table 1, which we refer to as “no default." In this variant, the interval of interest rate multiplicity spans the range of debt where the policy functions are below the 45 degree line and, as a result, the government pulls back. The points of intersection of the policy functions with the 45 degree line lie completely outside of the region of multiplicity, and the government never enters the Cole-Kehoe crisis zone and never defaults. Consequently, the equilibrium bond spread is always zero.



(a) Baseline



(b) Exposed



(c) No default

Figure 6: Optimal borrowing and equilibrium interest rate in the combined model

C Quantifying implicit transfers for Spain

In this section, we provide a quantification of the value of implicit transfers received by Spain from the European Central Bank via the Public Sector Purchase Programme (PSPP). The bond purchases started in March 2015 and have continued for the next ten years. To the extent that the ECB's holding of Spanish bonds is motivated politically, the true risk associated with these assets is likely higher than what the observed interest rate reflects. In such case, the Spanish government receives an implicit transfer by being able to pay a lower interest rate on these bonds than in the counterfactual scenario of no such intervention. Figure 7 illustrates the cumulative net purchases of Spanish government bonds over time, along with hypothetical unwinding forecasts. The purchases increased steadily in the first few years and the stock exceeded 0.25 trillion euro in 2018, falling back to this level only in 2025Q1. At the pace of the selloff that started from the peak in 2022, it would take at least another decade to unwind the entire portfolio.

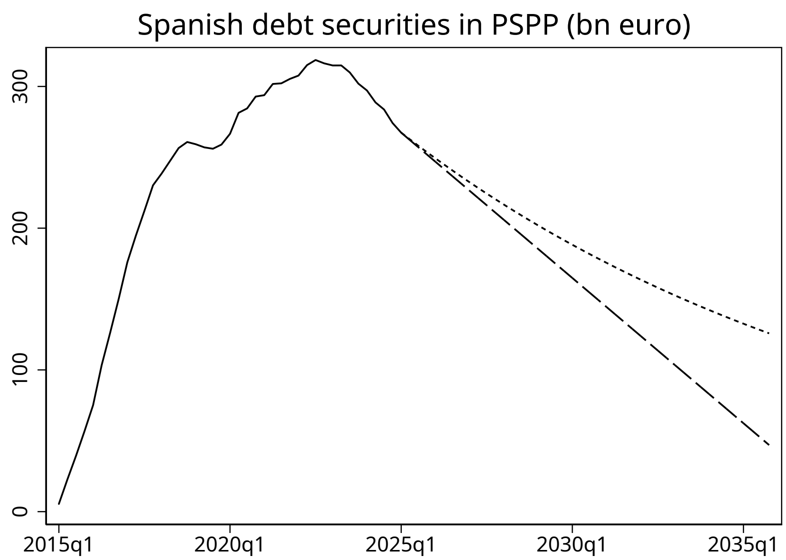


Figure 7: Cumulative net purchases for Spanish bonds under PSPP

Data source: https://www.ecb.europa.eu/mopo/implement/app/html/index.en.html#pspp. The forecasts beyond 2025Q1 are the authors' own calculations derived by extrapolating the linear and geometric trends from the peak in 2022Q3.

To calculate the implicit transfer, we first define a parameter κ that captures the interest rate differential associated with the intervention. That is, assume that in the absence of the ECB's bond purchase program the true Spanish spread would have been higher by a constant percentage amount κ. In other words,

$$\kappa \equiv \text{Yield differential absent ECB intervention}$$

Then we proceed as follows. We set κ to a fixed percentage amount. Next, for each quarter, we calculate the implicit transfer resulting from lower interest rate by multiplying κ by the outstanding amount of Spanish bonds in the ECB portfolio. Finally, we calculate the present value of all these transfers in 2015Q1 (using the German government bond yield as risk-free rate), we sum them up and divide by the Spanish GDP.

Table 2: Implicit transfer to Spain by assumed value of κ (% of 2015 GDP)

κ **(in %)**	**0.1**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
end sample at 2025Q1	0.22	1.11	2.22	3.33	4.43	5.54
linear extrap. to 2038Q1	0.36	1.79	3.57	5.36	7.14	8.93
geometric extrap. to 2038Q1	0.40	1.98	3.96	5.94	7.92	9.90

Note: in extrapolating series, we assume the risk-free interest rate in all periods beyond 2025Q1 is the average rate over 2022Q3-2025Q1 equal to 2.27%.

Table 2 presents the results of our calculation for different values of κ, and for different assumptions about the unwinding of the ECB-held bonds beyond the latest data observation (2025Q1). When we only consider the actually observed data (that is, we assume that ECB sells all Spanish bonds in 2025Q1), the total implicit transfer varies between 0.2% and 5.5% of Spain's GDP in 2015 for κ ranging between 0.1% and 2.5%. For reference, the average spread on Spanish bonds observed between 2015Q1 and 2025Q1 was 1.03%, while the maximum spread topped at 5.07% in 2012Q3. Hence, this seems like a reasonable range of ECB-induced yield differentials. On the other hand, if we extrapolate the bond holdings into the future (given that in 2025Q1 the ECB still held over 0.25 trillion euro worth of Spanish bonds as Figure 7 shows), the total value of the implicit transfer ranges from 0.4% to around 9-10% of 2015 GDP depending on the extrapolation method.

Naturally, the calculation above is designed to illustrate magnitudes and should be taken with a grain of salt. On the one hand, we derive the implicit transfer only from the bonds actually held by the ECB and ignore any spillover on the interest rate on the bonds held by private investors (by increasing confidence in the market). On the other hand, large asset purchases may also have generated moral hazard on the part of the Spanish government, inducing it to issue excessive debt (especially during the Covid-19 episode). Ultimately, we ignore these hard-to-quantify forces that work in opposite directions and focus on what we can observe directly.

D Calibration for Mexico

In this section, we calibrate our infinite-horizon model to Mexico. The purpose of this exercise is to show the broader range of outcomes that can be obtained with our model, especially for the case of an emerging market economy. Mexico is an instructive reference point as it is often use by related studies in the literature (Aguiar et al., 2022).

Table 3: Simulation results in the quantitative model calibrated to Mexico

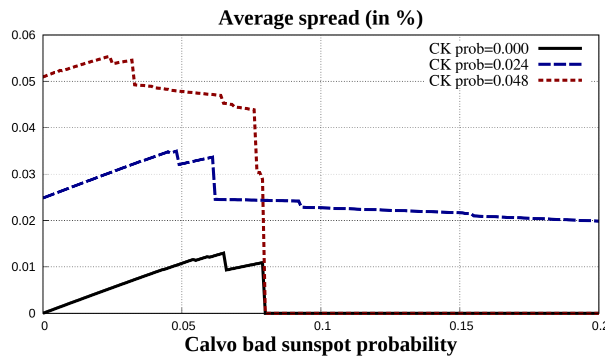
Statistic	**Model with growth regimes**		**Data**
	(1)	(2)	(3)
$P(s_c = 0)$	0.17	0.00	
$P(s_{ck} = 0)$	0.04	0.00	
avg(A/Y)	16.5%	33.9%	16.5%
avg$(spreads)$	3.0%	0.0%	3.0%
std$(spreads)$	2.0%	0.0%	2.2%
corr$(spreads, TB)$	-0.28	0	0.60
corr(TB, Y)	-0.19	-0.68	-0.40
corr$(spreads, Y)$	-0.01	0	-0.21

Note: $P(s_{ck} = 0)$ and $P(s_c = 0)$ denote the probabilities of a bad Cole-Kehoe-type and Calvo-type sunspot realization, respectively. Y = output, A = debt service, TB = trade balance-to-output ratio. All statistics are computed over non-default periods.

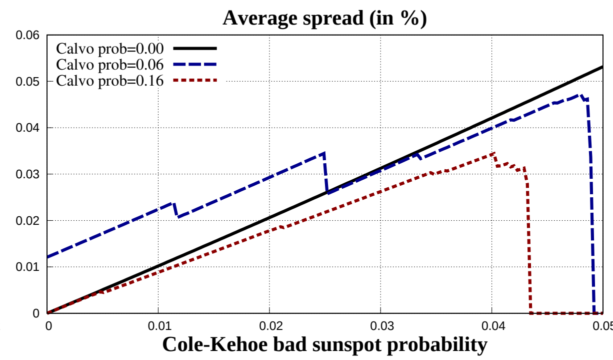
Consider the same model as in Section 3.4. We first estimate the income process for Mexico with its GDP data in years 1980-2021. We use the filter of Kim (1994) and estimate the parameters with maximum likelihood. The resulting estimates are $g_h = 1.02$, $g_l = 0.96$, $\pi_H = 0.8$ and $\pi_L = 0.3$. Parameters r, γ and θ are kept at their values of 0.01, 2, and 0.125, respectively. The discount factor β is set to 0.8 following Aguiar et al. (2022). We then proceed to calibrate the default cost ϕ, along with the two sunspot probabilities, to match the three moments of interest, namely the average and standard deviation of the interest rate spread and the average debt-to-income ratio. The inferred parameter values are $\phi = 3.25\%$, $p_c = P(s_c = 1) = 0.17$, and $p_{ck} \equiv P(s_{ck} = 1) = 0.04$. Hence, relative to the calibration for Spain, Mexico requires a combination of lower discount factor and higher default cost, along with a lower probability of a Calvo crisis and higher probability of a rollover crisis.

Columns (1) and (3) in Table 3 show that the model matches the average debt and interest rate spread very well, and it comes quite close to getting the full extent of the volatility of the spread. This is a considerably different calibration from Spain which involves a lower value for each of these moments. The model also delivers a mildly countercyclical spread. As in the case of Spain, column (2) shows that sunspot realizations are essential for generating the quantitative results, consistent with the findings of Aguiar et al. (2022).

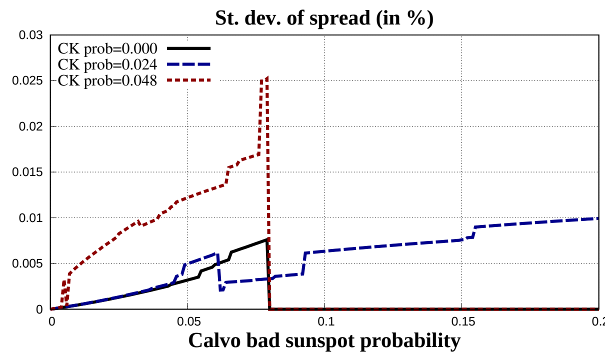
We also replicate the comparative statics analysis as in Section 3.2 for the case of Mexico. Panels 8(a), 8(c), and 8(e) of Figure 8 plot the three key moments while varying the bad Calvo sunspot probability up to 0.2, for three different values of the bad Cole-Kehoe sunspot probability, while panels 8(b), 8(d), and 8(f) do the opposite. We observe a similar pattern as for the case of Spain. By varying the probabilities of bad realization for both sunspots, our model generates a wide range of combinations of the debt and spread moments than the model variant with a single source of multiplicity (black solid line in the graphs). In particular, our model easily achieves a standard deviation of the spread between 2% and 2.5%, while also offering various combinations of average debt and average spread. Figure 8 also shows, similar as with our results for Spain, that the two sources of multiplicity deliver our results either by operating in isolation (for low enough probabilities of both bad Calvo and Cole-Kehoe sunspots), or by interacting and generating a richer dynamics than the sum of its parts.



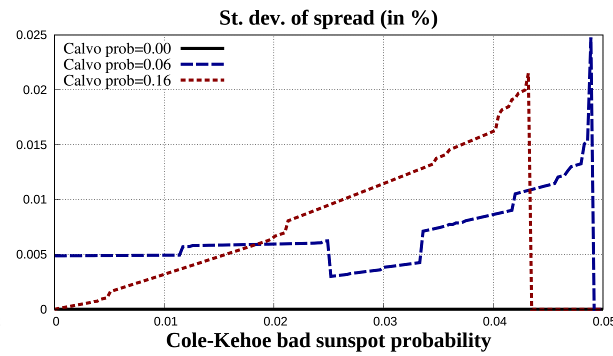
(a) Average spread



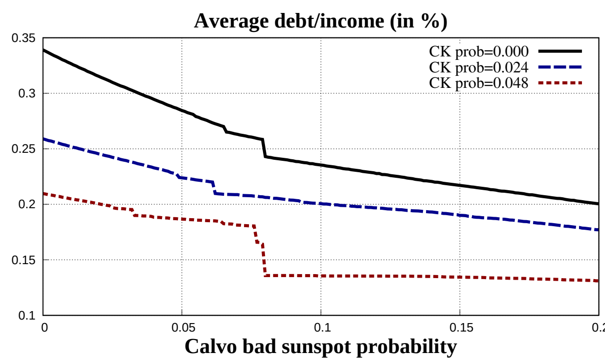
(b) Average spread



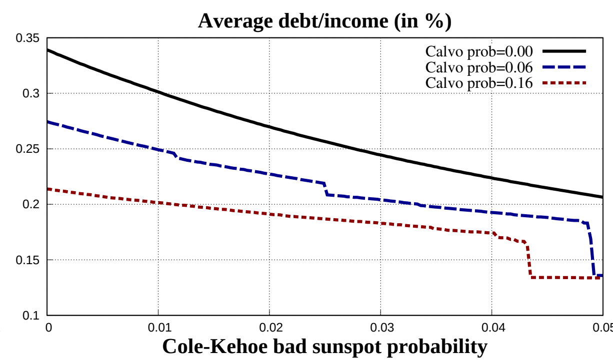
(c) Standard deviation of spread



(d) Standard deviation of spread



(e) Average debt



(f) Average debt

Figure 8: Comparative statics in the baseline calibration for Spain