Sovereign Default: The Role of Expectations

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- Can self-fulfilling expectations play a role in sovereign debt crisis?

- Argentina defaulted in December 2001.

- Starting in 1991, a currency board was established

  1. Inflation was like in the US.

  2. GDP grew 50% over the decade.

  3. Average debt to GDP ratio was 40%. Deficit never above 2%.

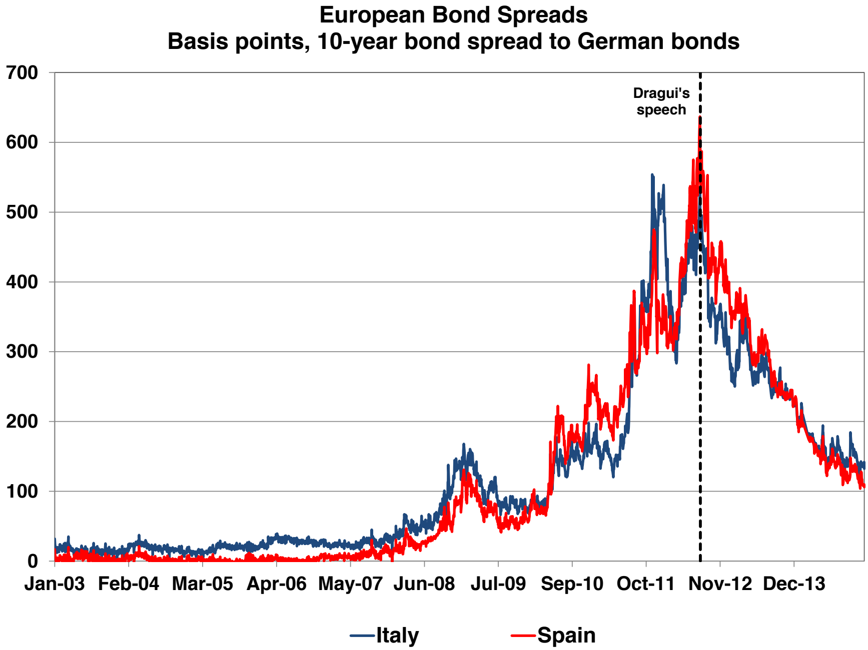
  4. It satified all conditions of Maastricht treaty each year...

.....except for the interest rate on government bonds. (Market determined)

- Average rate on dollar denominated bonds was $10\% = 4\% + 6\%$.

- Implied, over the decade, an additional payment of around 20% of GDP, half of the debt.

- Would Argentina had defaulted had the rates been 4%?



Source: Global Financial Data

- Can a country be trapped in an equilibrium with high interest rates because default probabilities are high, but default probabilities are high because interest rates are high?

- Can Policy rule out those equilibria? Did the ECB save Europe?

- Calvo (1988), more recently Lorenzoni and Werning (2013).

- Eaton-Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), compute a single low interest rate equilibrium.

- Cole-Kehoe (2000): maturity mismatch and rollover risk.

Theoretical contribution: We take a model similar to the one in Aguiar and Gopinath (2006) and Arellano (2008), and show that multiple equilibria arise with minor changes in modelling choices concerning

1. the timing of moves by single debtor and atomistic creditors or

2. the strategy space of debtor.

- The change in modelling choices are minor because there is no direct evidence to discriminate across them.

- Hard to rule out multiplicity on theoretical grounds.

Quantitative contribution: Numerically solve an infinite period model.

- Key ingredient: Markov switching on growth rates of output.
- Substantial likelihood of a prolonged stagnation.

Plan

- Very simple two period model: Two equilibria, one with low rates, one with high rates.

- Role for policy.

- Alternative timing and action assumptions. Literature.

- Dynamic model with sunspot variable that selects schedules. Numerical exercise.

The two-period small open economy model

- Borrower is a small open economy with preferences given by

$$U(c_1) + \beta E U(c_2)$$

- Income process

$$\begin{aligned} y_1 &= 1 \\ y_2 &= \begin{cases} y^l, & \text{probability } p \\ y^h, & \text{probability } (1-p) \end{cases} \\ 1 &< y^l < y^h \end{aligned}$$

- Continuum of risk neutral foreign lenders.

- Alternative return is R^*.

- The expected return of lending to the country, taking default into account, R, has to be equal to R^*.

- Uncontingent bond. Zero initial debt.

- Borrow b, at a rate R

  1. pay $bR = a$.

  2. default.

- If default, output is lost and $c_2 = 1$.

- No recovery value.

- Timing: who moves first?

- Action chosen by borrower:

  1. b, amount received at $t = 1$.

  2. $a = bR$, amount paid at $t = 2$.

Action

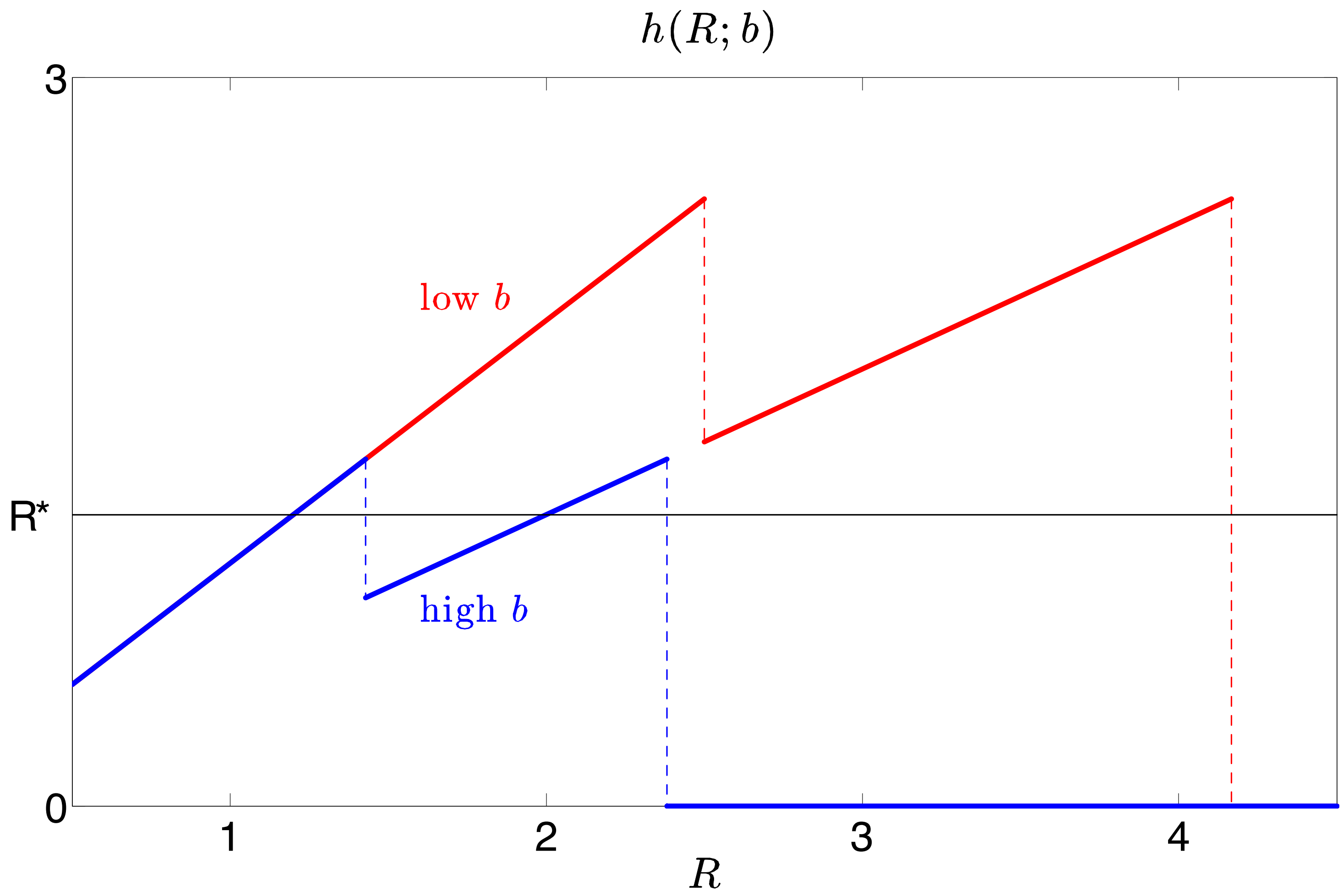
		Current debt b	Debt at maturity a
First Mover	Lenders		
	Borrower		

- First period:
  - Creditor $i \in [0, 1]$ offers limited funds at gross interest rate R_i.
  - The borrower moves next and borrows from the low rate creditors $b = \int_0^1 b_i di$.
- In equilibrium, $R_i = R$. Let $b_i = b$.
- Second period: The borrower defaults if cost of paying is larger than benefit of paying

$$bR < y_2 - 1.$$

- Given a value for b, the expected return for lenders is

$$h(R;b) = \begin{cases} R, & \text{if } Rb \leq \left(y^l - 1\right) \\ \\ R\ (1-p), & \text{if } \left(y^l - 1\right) < bR \leq \left(y^h - 1\right) \\ \\ 0, & \text{if } Rb > \left(y^h - 1\right). \end{cases}$$

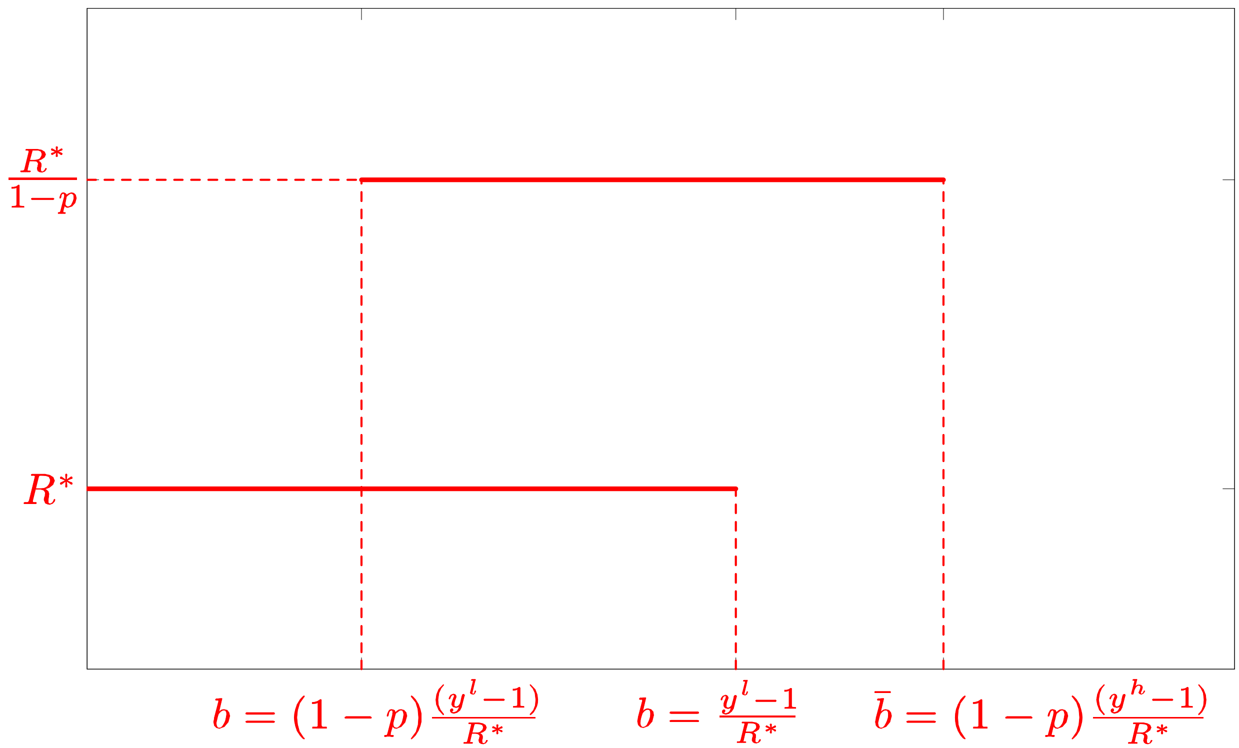


$h(R;b)$
3
R*
0
1
2
3
4
R
low b
high b

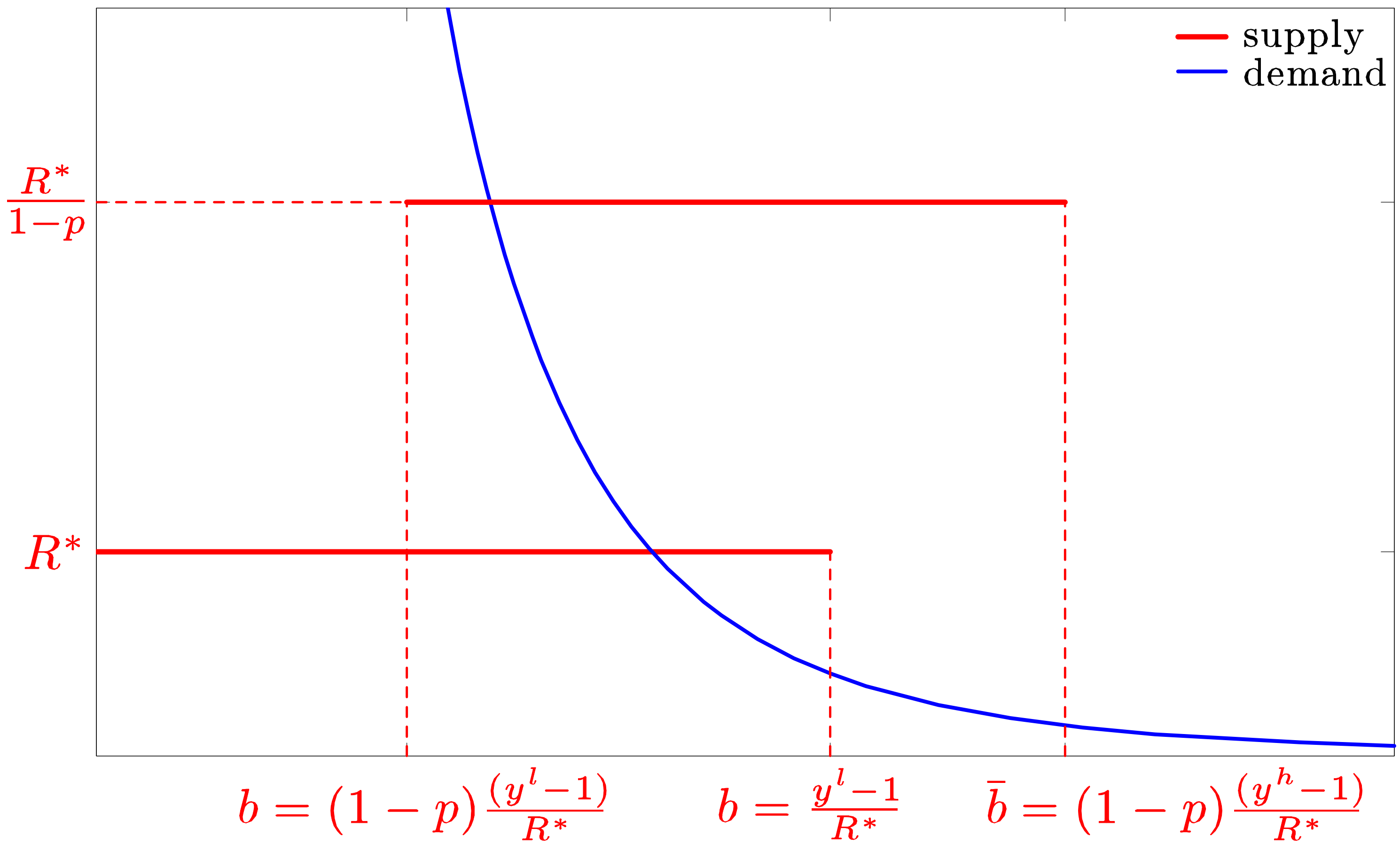
Supply

$$R(b) = \begin{cases} R^*, & \text{if } R^* b \leq y^l - 1 \\ \frac{R^*}{1-p}, & \text{if } y^l - 1 < \frac{R^*}{1-p} b \leq y^h - 1 \\ \infty, & \text{if } \frac{R^*}{1-p} b > y^h - 1 \end{cases}$$

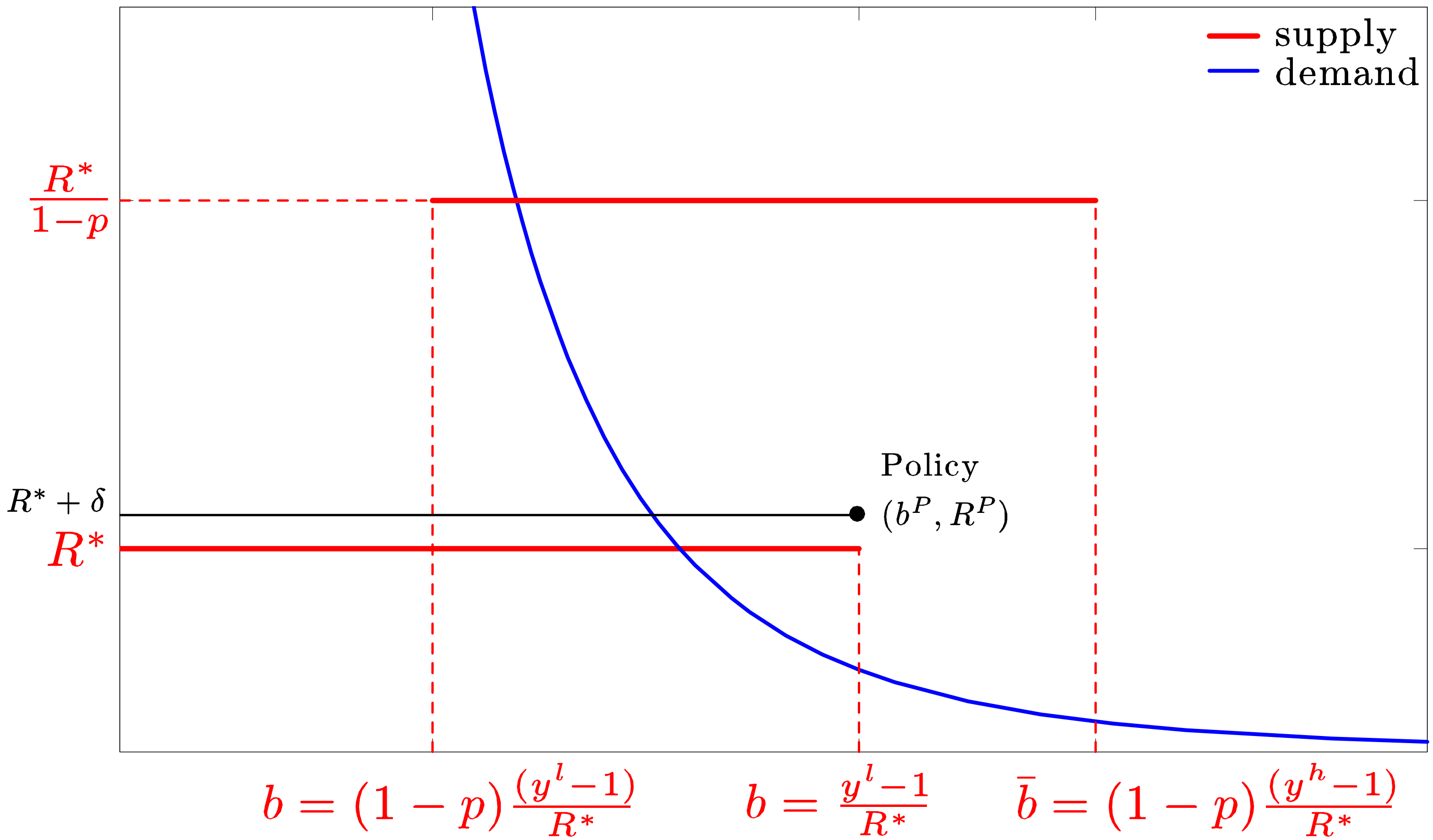
Thus, the supply is given by



so, given a high enough demand, there may be two equilibria.


supply
demand
$\frac{R^*}{1-p}$
R^*
$b = (1-p)\frac{(y^l-1)}{R^*}$
$b = \frac{y^l-1}{R^*}$
$\bar{b} = (1-p)\frac{(y^h-1)}{R^*}$

Policy



supply
demand
$\frac{R^*}{1-p}$
$R^* + \delta$
R^*
Policy
(b^P, R^P)
$b = (1-p)\frac{(y^l-1)}{R^*}$
$b = \frac{y^l-1}{R^*}$
$\bar{b} = (1-p)\frac{(y^h-1)}{R^*}$

Action

		Current debt b	Debt at maturity a
First Mover	Lenders	Multiplicity	?
	Borrower		

Alternative action

- Does it matter (for multiplicity) whether the choice for the borrower is b or $a = Rb$?

  1. Calvo (1988). Schedule in b, multiple equilibria.

  2. Aguiar and Gopinath (2006), Arellano (2008). Schedule in a, single equilibrium.

- For lenders, it is clearly inessential.

Demand

$$\max_{b} u(\omega + b) + \beta \left\{ \begin{array}{c} pu\left(\max\left\{y^L - Rb, 1\right\}\right) + \\ (1-p)\,u\left(\max\left\{y^L - Rb, 1\right\}\right) \end{array} \right\}$$

$$s.t. \quad Rb \leq y^H - 1$$

Given R, if b^* is a solution to the borrower problem when choosing b, then $a^* \equiv Rb^*$ is a solution to the borrower when choosing a.

Demand

$$\max_{a} u\left(\omega + \frac{a}{R}\right) + \beta \left\{ \begin{array}{c} pu\left(\max\left\{y^L - a, 1\right\}\right) + \\ \\ (1-p)\,u\left(\max\left\{y^H - a, 1\right\}\right) \end{array} \right\}$$

$$s.t. \quad a \leq y^H - 1$$

Given R, if b^* is a solution to the borrower problem when choosing b, then $a^* \equiv Rb^*$ is a solution to the borrower when choosing a.

		Action	
		Current debt b	Debt at maturity a
First Mover	Lenders	Multiplicity	Multiplicity
	Borrower	?	?

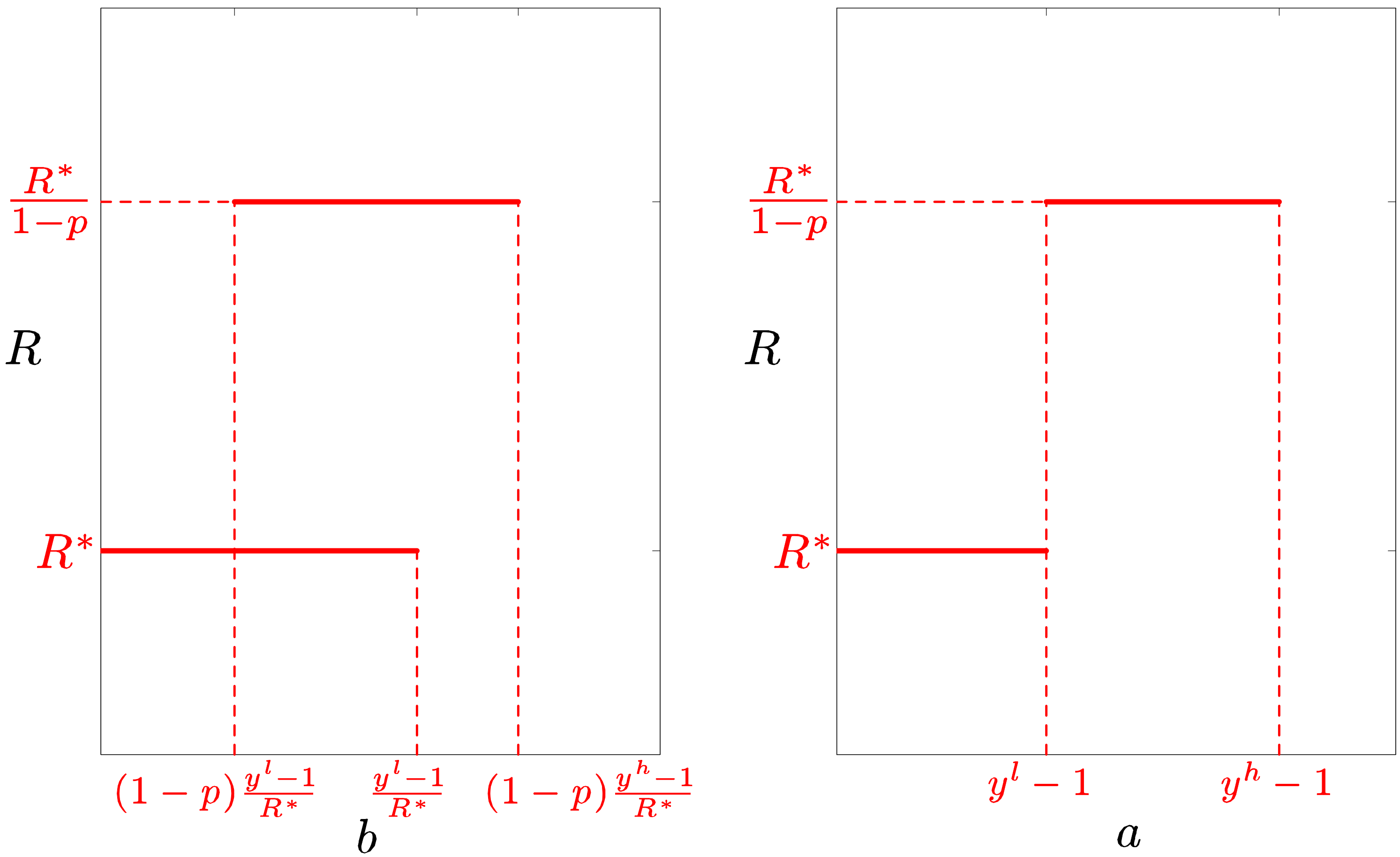
Alternative timing/action

- The borrower moves first and chooses b or $a = bR$.

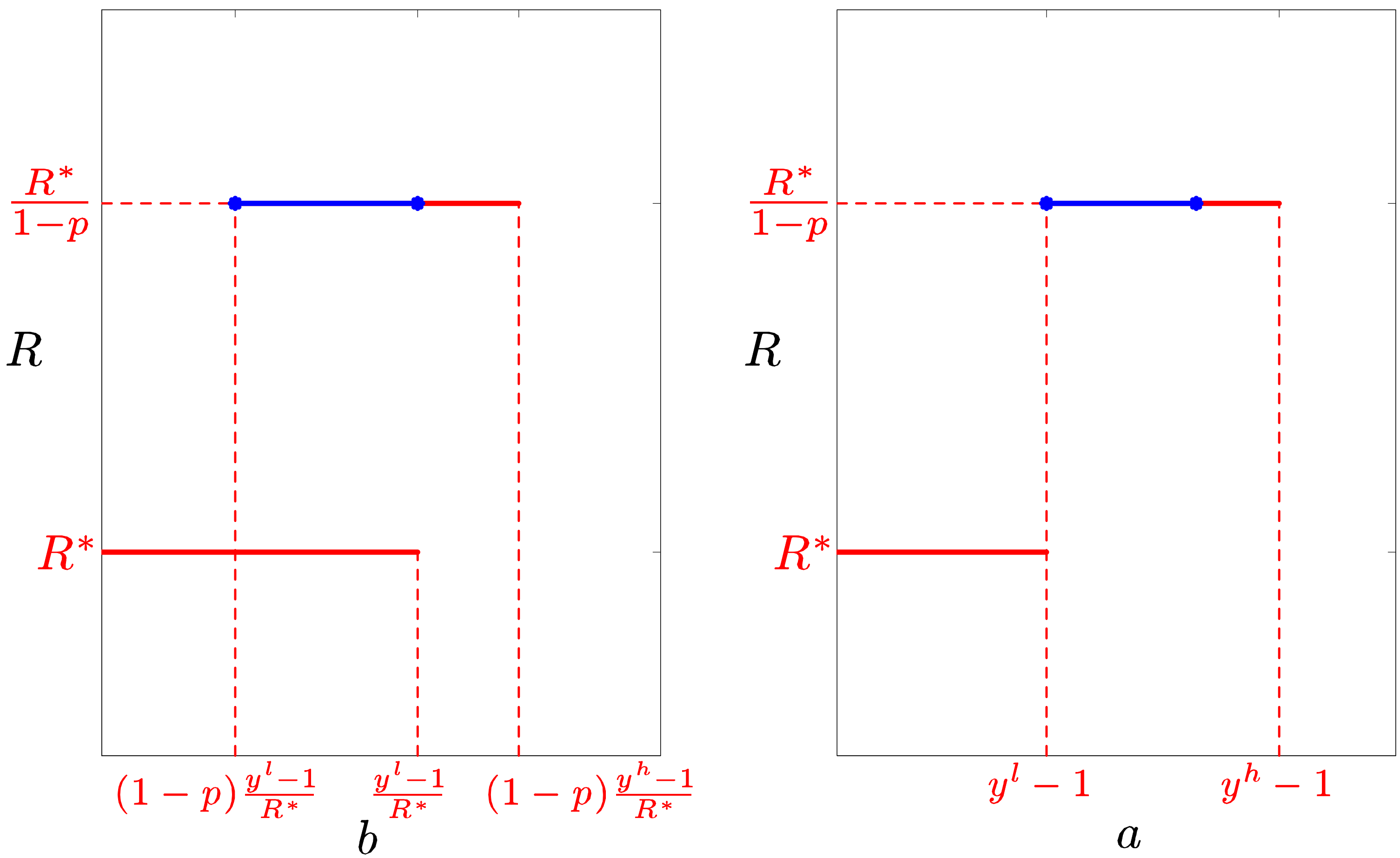
- If the choice is b or a, creditors move next and offer *schedules* $R(b)$ or $q(a) = \frac{1}{R(a)}$.

$$R(b) = \begin{cases} R^*, & \text{if } R^* b \leq \left(y^l - 1\right) \\ \frac{R^*}{1-p}, & \text{if } \left(y^l - 1\right) < \frac{R^*}{1-p} b \leq \left(y^h - 1\right) \\ \infty, & \text{if } \frac{R^*}{1-p} b > \left(y^h - 1\right) \end{cases}$$

$$R(a) = \begin{cases} R^*, & \text{if } a \leq \left(y^l - 1\right) \\ \frac{R^*}{1-p}, & \text{if } \left(y^l - 1\right) < a \leq \left(y^h - 1\right) \\ \infty, & \text{if } a > \left(y^h - 1\right) \end{cases}$$



$\frac{R^*}{1-p}$
R
R^*
$(1-p)\frac{y^l-1}{R^*}$
$\frac{y^l-1}{R^*}$
$(1-p)\frac{y^h-1}{R^*}$
b
$\frac{R^*}{1-p}$
R
R^*
y^l-1
y^h-1
a



$\frac{R^*}{1-p}$
R
R^*
$(1-p)\frac{y^l-1}{R^*}$
$\frac{y^l-1}{R^*}$
$(1-p)\frac{y^h-1}{R^*}$
b
$\frac{R^*}{1-p}$
R
R^*
y^l-1
y^h-1
a

For a general density $f(y)$ and CDF $F(y)$,

- If the borrower chooses b,the schedule is

$$R^{*} = R\left[1 - F\left(1 + Rb\right)\right]$$

- If the borrower chooses a, the schedule is

$$R^{*} = R\left[1 - F\left(1 + a\right)\right]$$

- Picking a is like picking the probability of default, or R.

		Action	
		Current debt b	Debt at maturity a
First Mover	Lenders	Multiplicity	Multiplicity
	Borrower	Multiplicity	Uniqueness

Lorenzoni-Werning (2013) (closer to Calvo (1988)).

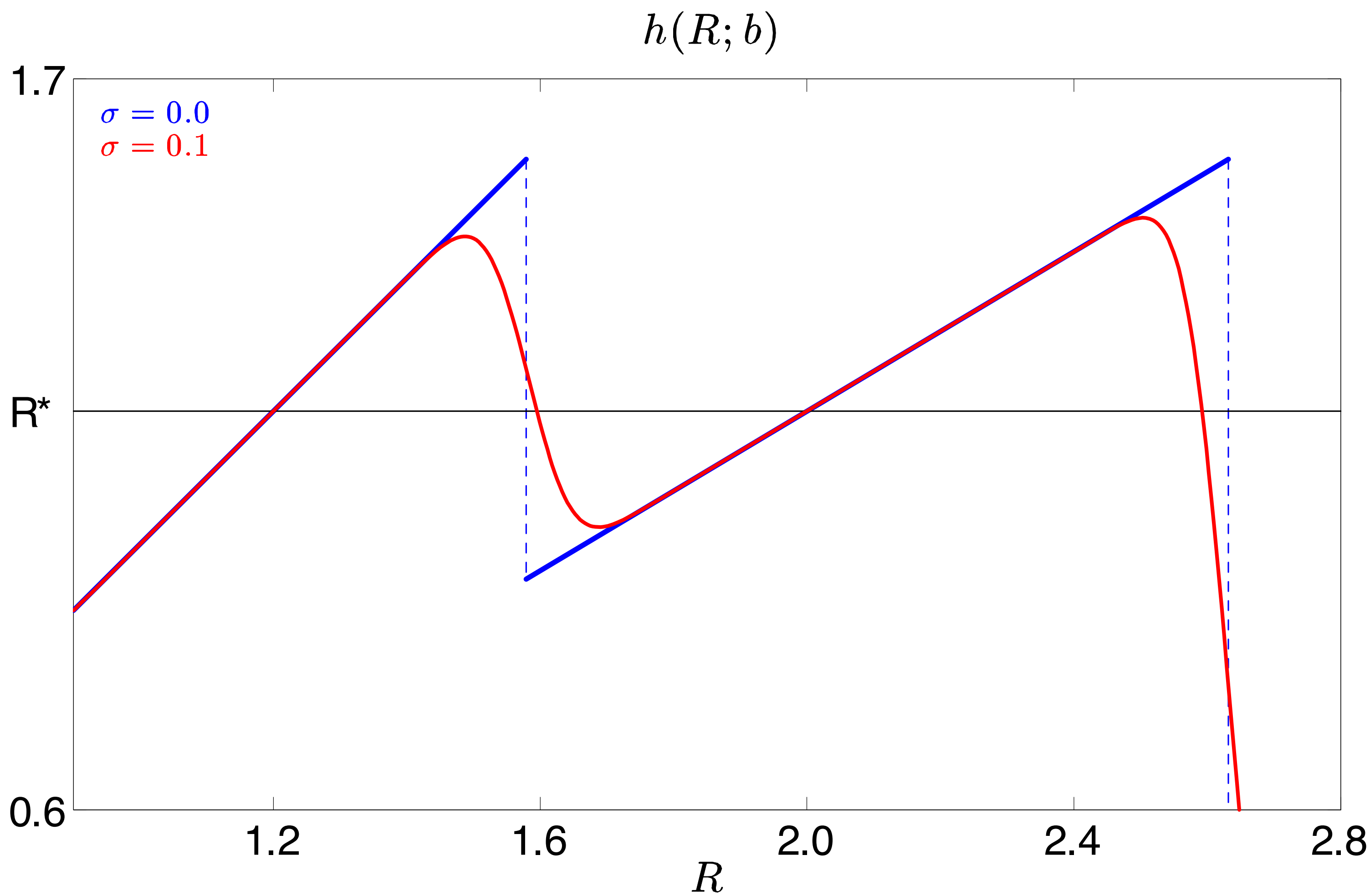
- The borrower is a government with exogenous deficits or surpluses. If the surplus is high enough the debt is repaid. Otherwise there is default.

- Argument against choice of a: Game without commitment within period.

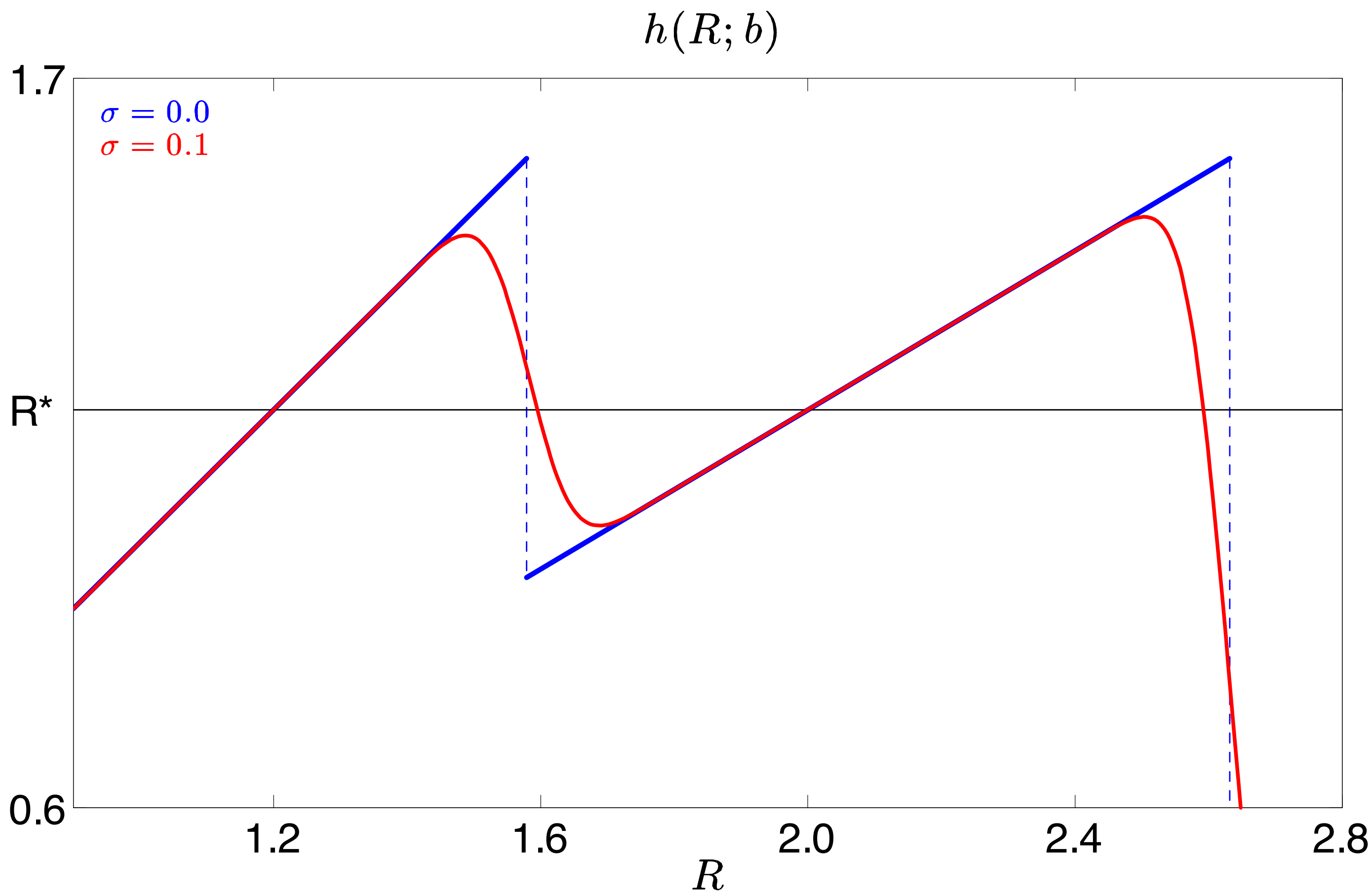
  - The government cannot commit not to reissue.

  - In the limit, government is a price taker (durable good monopoly).

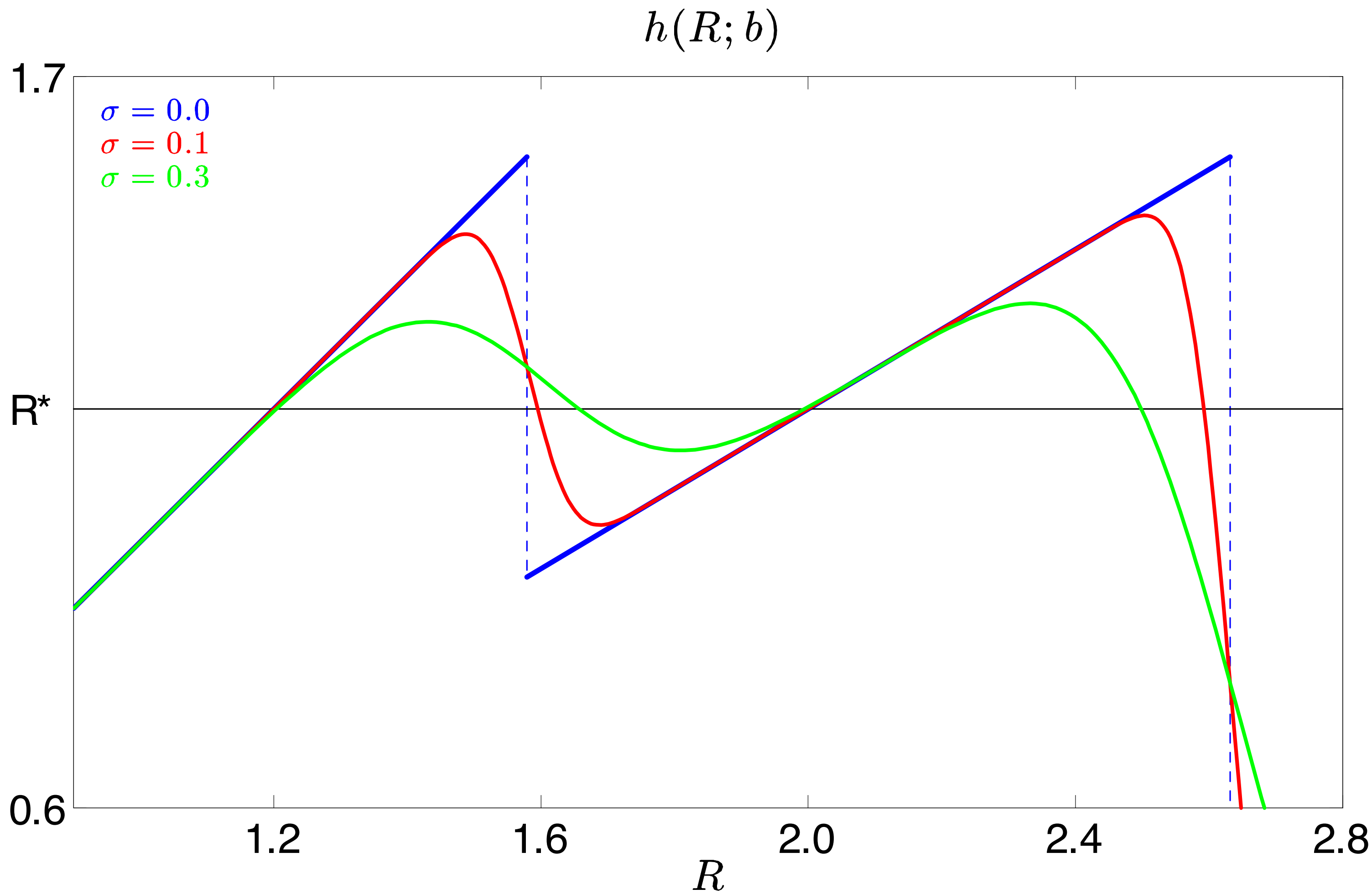
- LW focus on equilibria with debt dilution. Longer maturity.



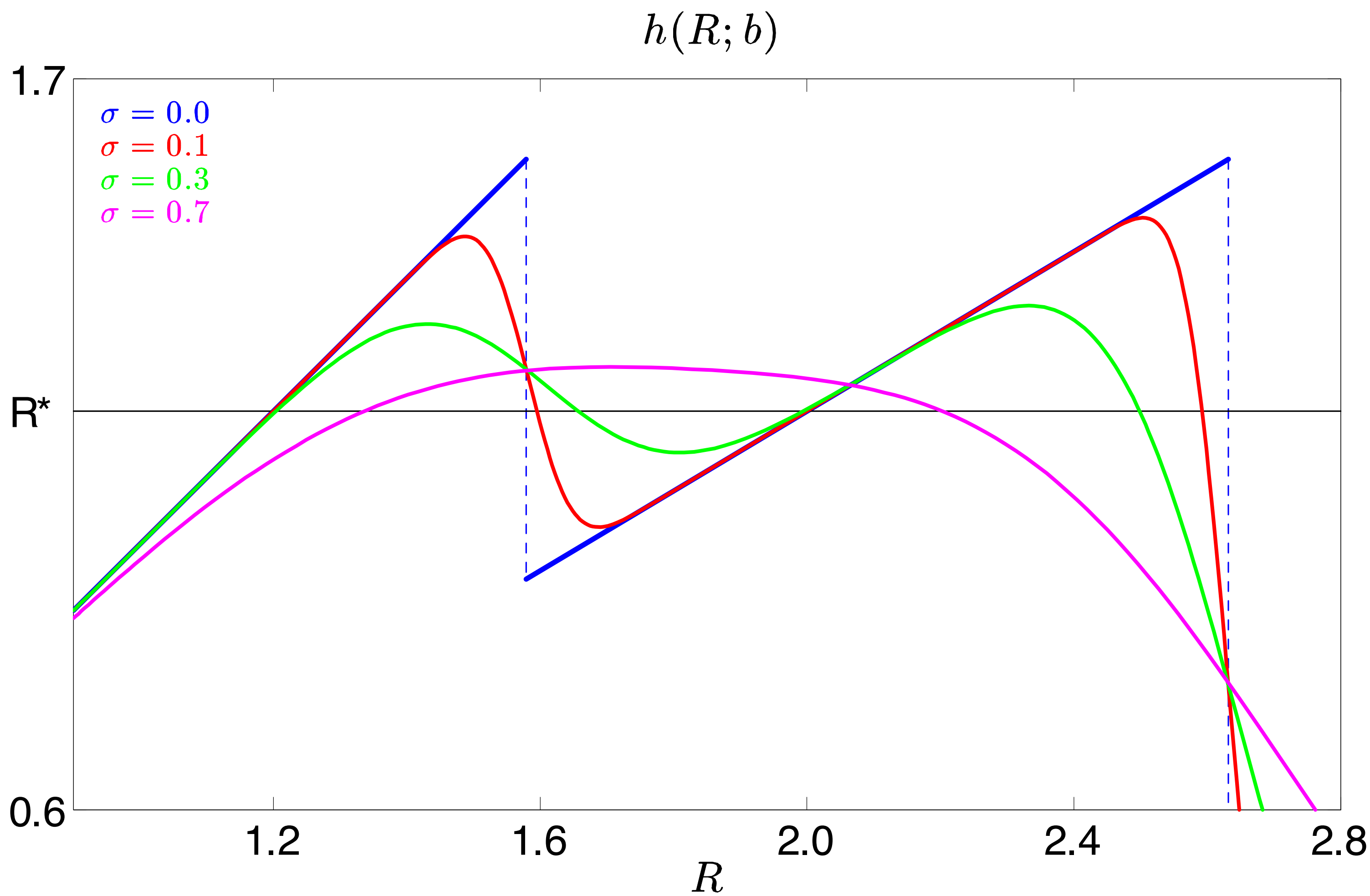
$h(R;b)$
1.7
$\sigma = 0.0$
$\sigma = 0.1$
R*
0.6
1.2
1.6
2.0
2.4
2.8
R



- Fragility of the drecreasing schedule.



$h(R;b)$
1.7
$\sigma = 0.0$
$\sigma = 0.1$
$\sigma = 0.3$
R*
0.6
1.2
1.6
2.0
2.4
2.8
R

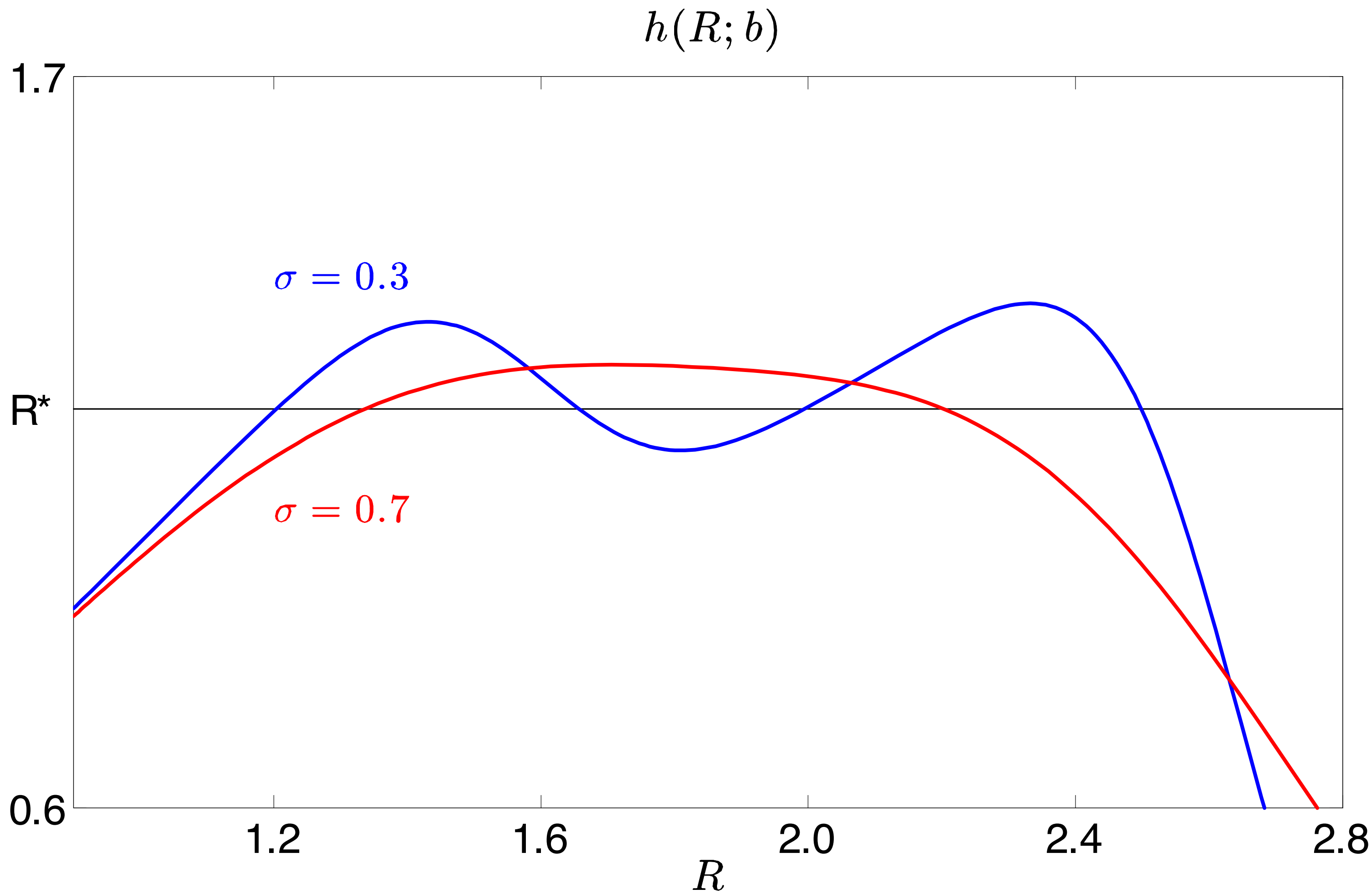


$h(R; b)$
1.7
R*
0.6
$\sigma = 0.0$
$\sigma = 0.1$
$\sigma = 0.3$
$\sigma = 0.7$
1.2
1.6
2.0
2.4
2.8
R

- Why crisis in Europe started at the end of 2009?

- Debt accumulation between 2009 and 2011 (3 deficits) was 72% to 108% for Portugal, 40% to 70% for Spain, and 106% to 120% for Italy.

- High growth in 90's, low growth in 2000's. Another decade of stagnation for the 2010's?



$h(R;b)$
1.7
R*
0.6
$\sigma = 0.3$
$\sigma = 0.7$
1.2
1.6
2.0
2.4
2.8
R

Dynamic model: Simulating sovereign debt crises.

- Government moves first and chooses b: Multiplicity.

- The endowment y has a bimodal distribution with cdf $F(y)$

- Upon default, utility is

$$V^{aut} = \frac{U(y^d)}{1-\beta}$$

- Sunspot $s = 1, 2$, used to select one of two (increasing) interest rate schedules.

- p is the probability of $s = 1$.

- Interest rate schedule: $R(b, s)$.

- Arbitrage conditions for the risk free creditors, in each state,

$$
\begin{aligned}
R^* &= R(b,1)\left[p\left(1 - F\left(\underline{y}(b,1,1)\right)\right) + (1-p)\left(1 - F\left(\underline{y}(b,1,2)\right)\right)\right] \\
R^* &= R(b,2)\left[p\left(1 - F\left(\underline{y}(b,2,1)\right)\right) + (1-p)\left(1 - F\left(\underline{y}(b,2,2)\right)\right)\right]
\end{aligned}
$$

For $j = 1, 2$

$$
\begin{aligned}
V(\omega, j) &= \max_{c,b,\omega'} \left\{ U(c) + \beta \mathbb{E}_{y'} \left[\begin{array}{c} p \max \left\{ V(\omega', 1), V^{aut} \right\} + \\ (1-p) \max \left\{ V(\omega', 2), V^{aut} \right\} \end{array} \right] \right\} \\
&\quad \text{subject to} \\
c &\leq \omega + b; \\
\omega' &= y' - bR(b, j); \\
b &\leq b^{\max}
\end{aligned}
$$

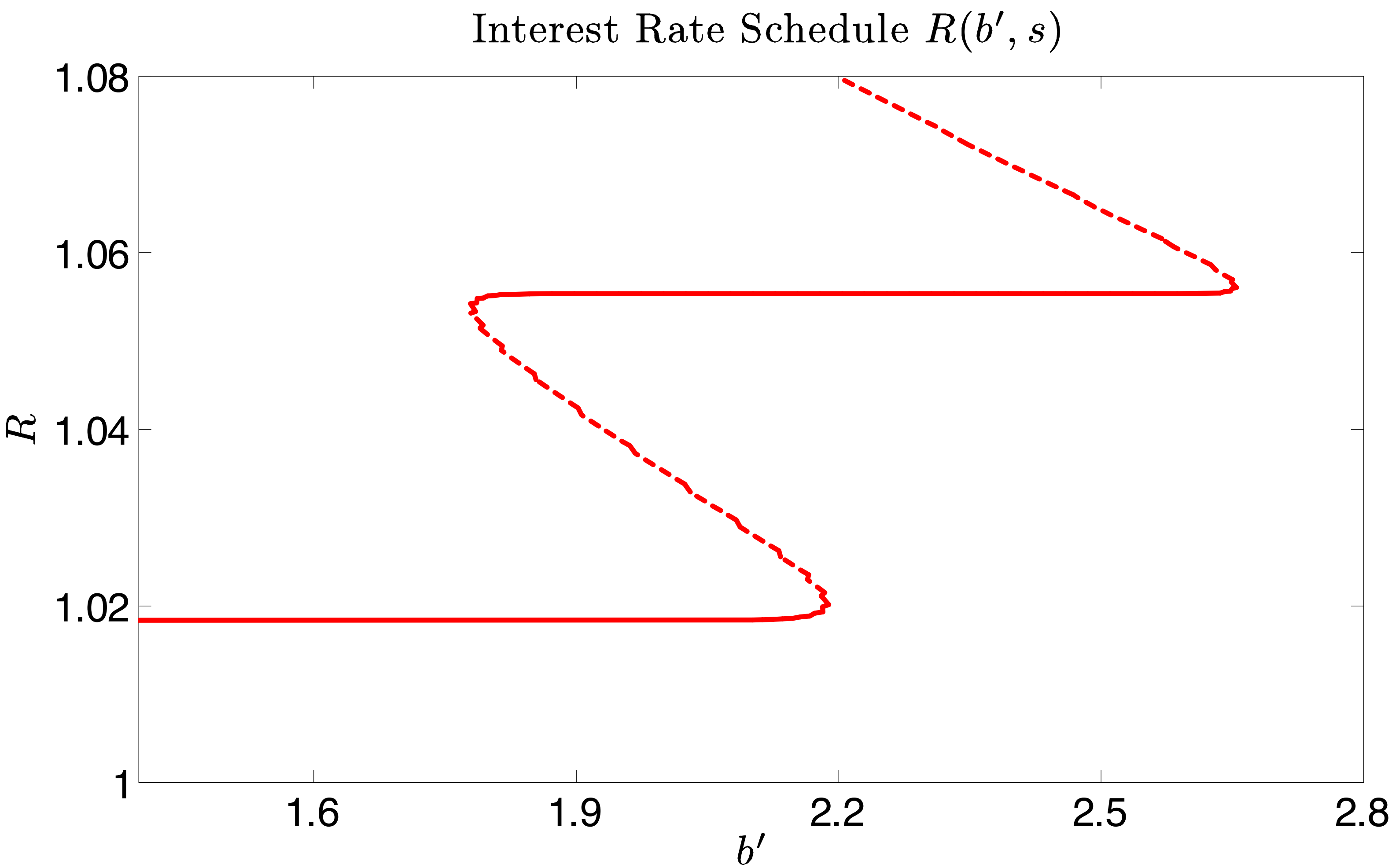
Parameter values

- Average maturity of government debt is about 8 years.
- Period is 10 years: $R^* = 1.2$, $\beta = 0.7$, $\gamma = 6$.
- Differences in estimated GDP growth rates between the high growth regime and low growth regime, between 3.5% and 4.5% a year.
- Equivalent to 42% to 55% growth over a decade.
- Endowment is drawn from one of two normals with mean 4 and 6, and a common standard deviation of 0.1.

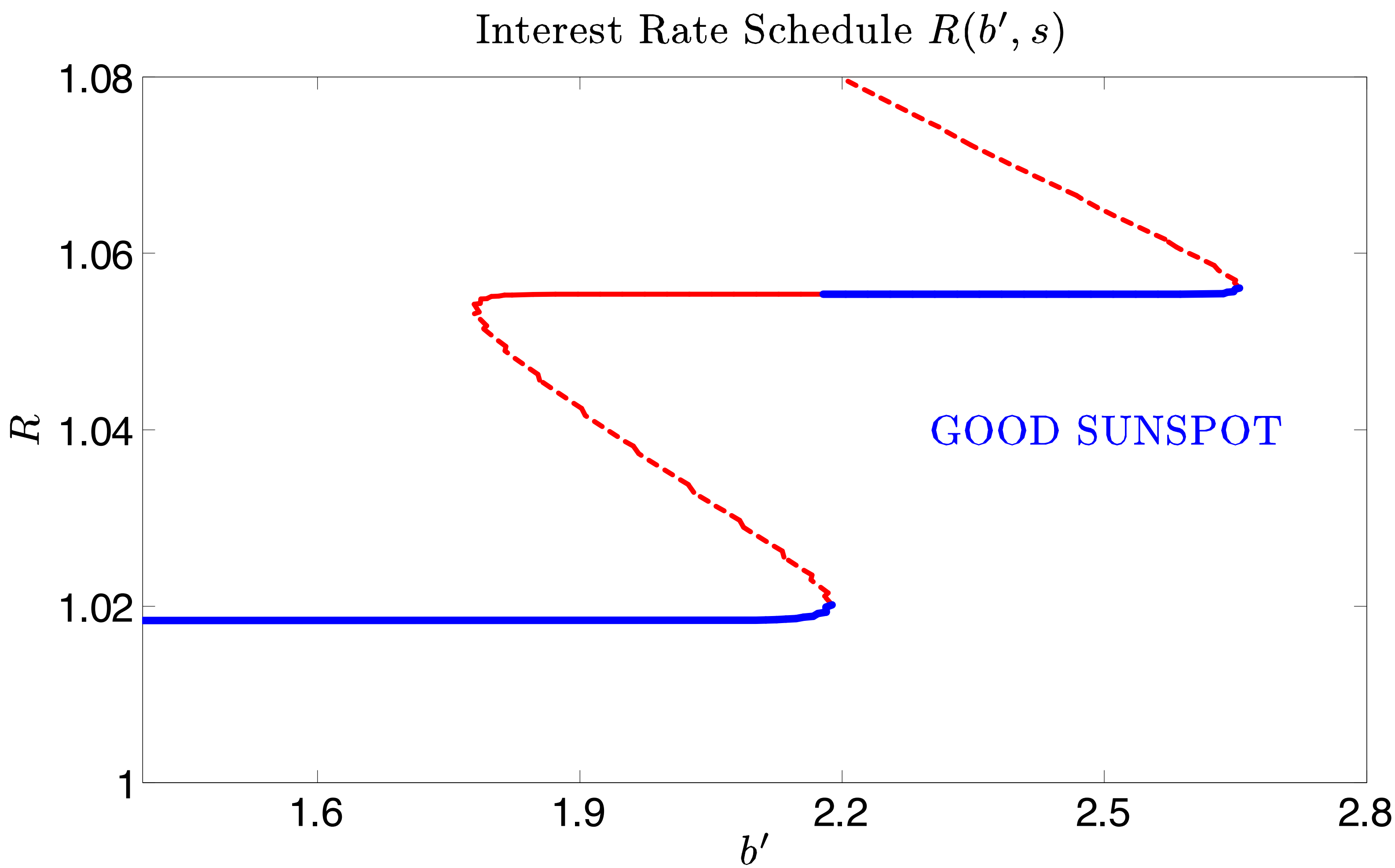
- Probability that nature chooses to draw from the bad distribution is $\pi = 0.3$.

- This shock is iid.

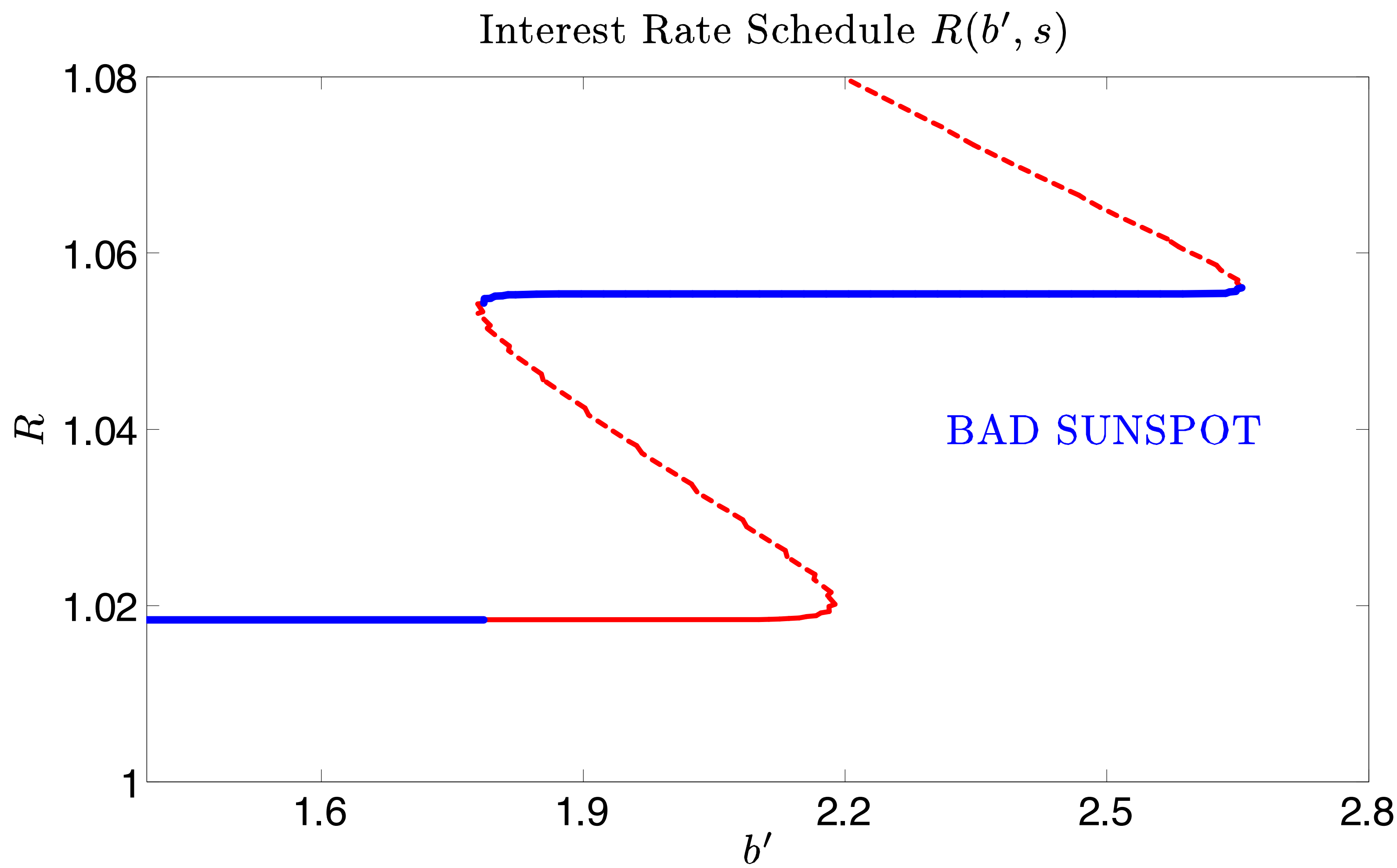
- Probability of the bad sunspot: 20%.



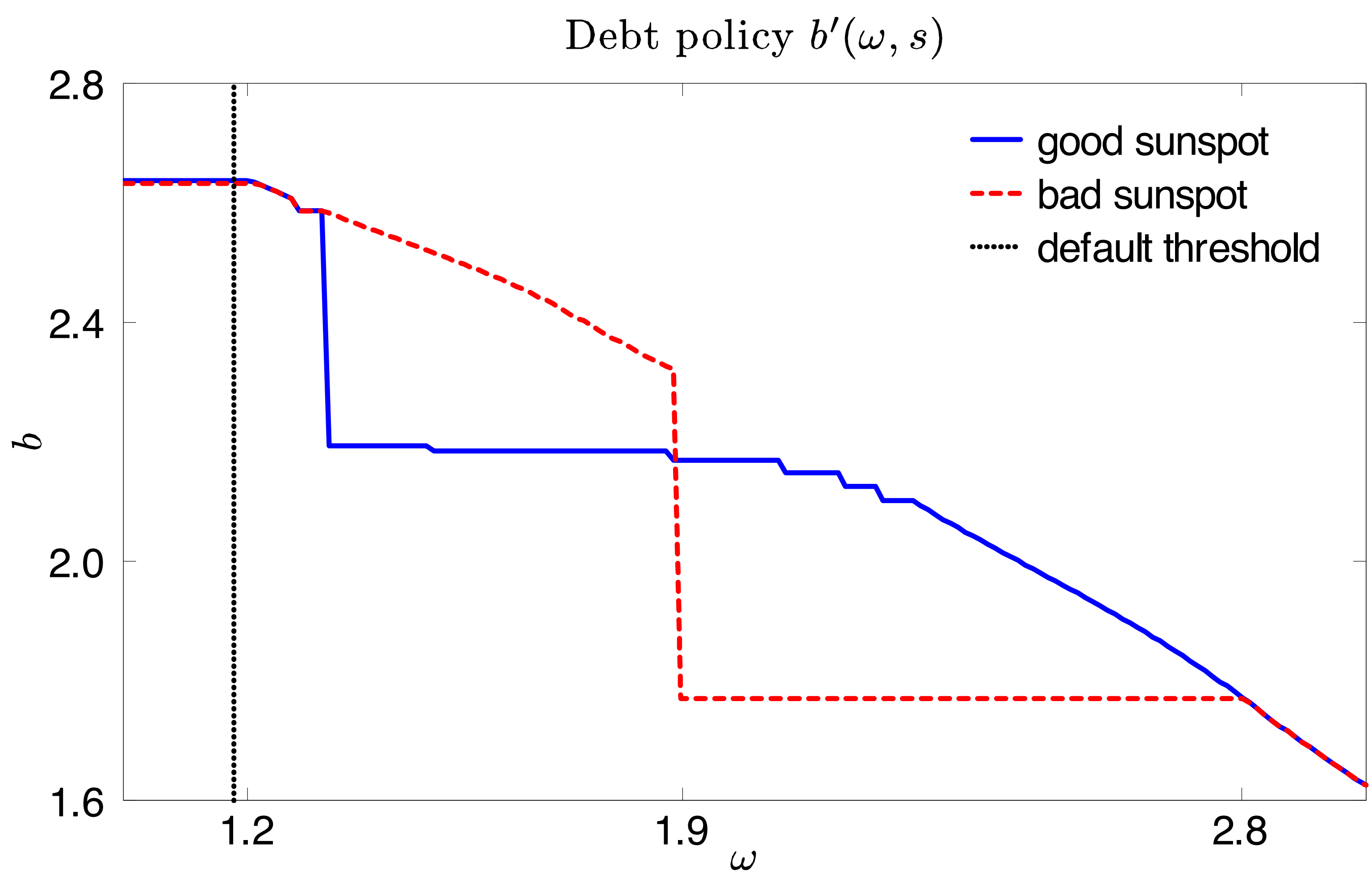
Interest Rate Schedule $R(b', s)$
1.08
1.06
1.04
1.02
1
R
1.6
1.9
2.2
2.5
2.8
b'



Interest Rate Schedule $R(b', s)$
1.08
1.06
1.04
1.02
1
R
1.6
1.9
2.2
2.5
2.8
b'
GOOD SUNSPOT



Interest Rate Schedule $R(b', s)$
1.08
1.06
1.04
1.02
1
R
1.6
1.9
2.2
2.5
2.8
b'
BAD SUNSPOT


Debt policy $b'(\omega, s)$
2.8
2.4
2.0
1.6
b
1.2
1.9
2.8
ω
good sunspot
bad sunspot
default threshold

Simulation of debt crisis

- We start the economy with wealth equal to 3.2 and assume the endowment is equal to 4 every period.
- Good sunspot realizes for 4 periods and then the bad sunspot realizes forever.
- In period period 11, the policy is implemented.
- It remains there forever.

Simulation of crisis
debt (left axis)
interest rate (right axis)
2.6
2.4
2.2
2
1.06
1.04
1.02
1
2
4
6
8
10
12
14
t

- Spreads jump up with the debt level.

- Policy can bring both spreads and debt down (austerity with policy).

- Relevance of multiplicity is endogenous. Hard to get borrower into the region of multiplicity.

Conclusion

- **Theory:** Hard to rule out multiplicity. Hard to get direct evidence.

- Indirect evidence?

- **Application:** Following 2008

  1. Large increases in public debt.

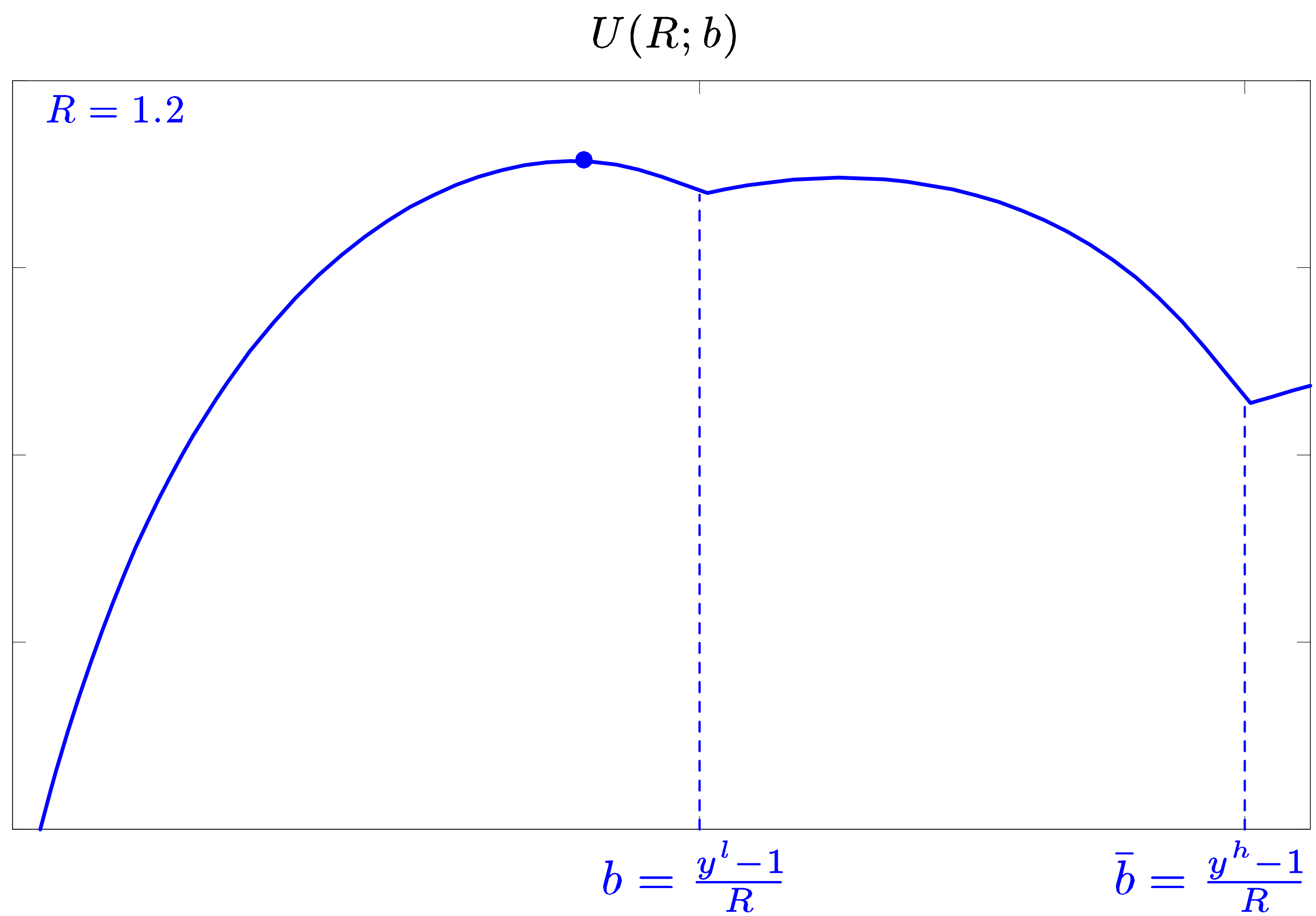
  2. Combined with a sizeable probabilty of a long stagnation.

- Draghi or German Constitutional Court?

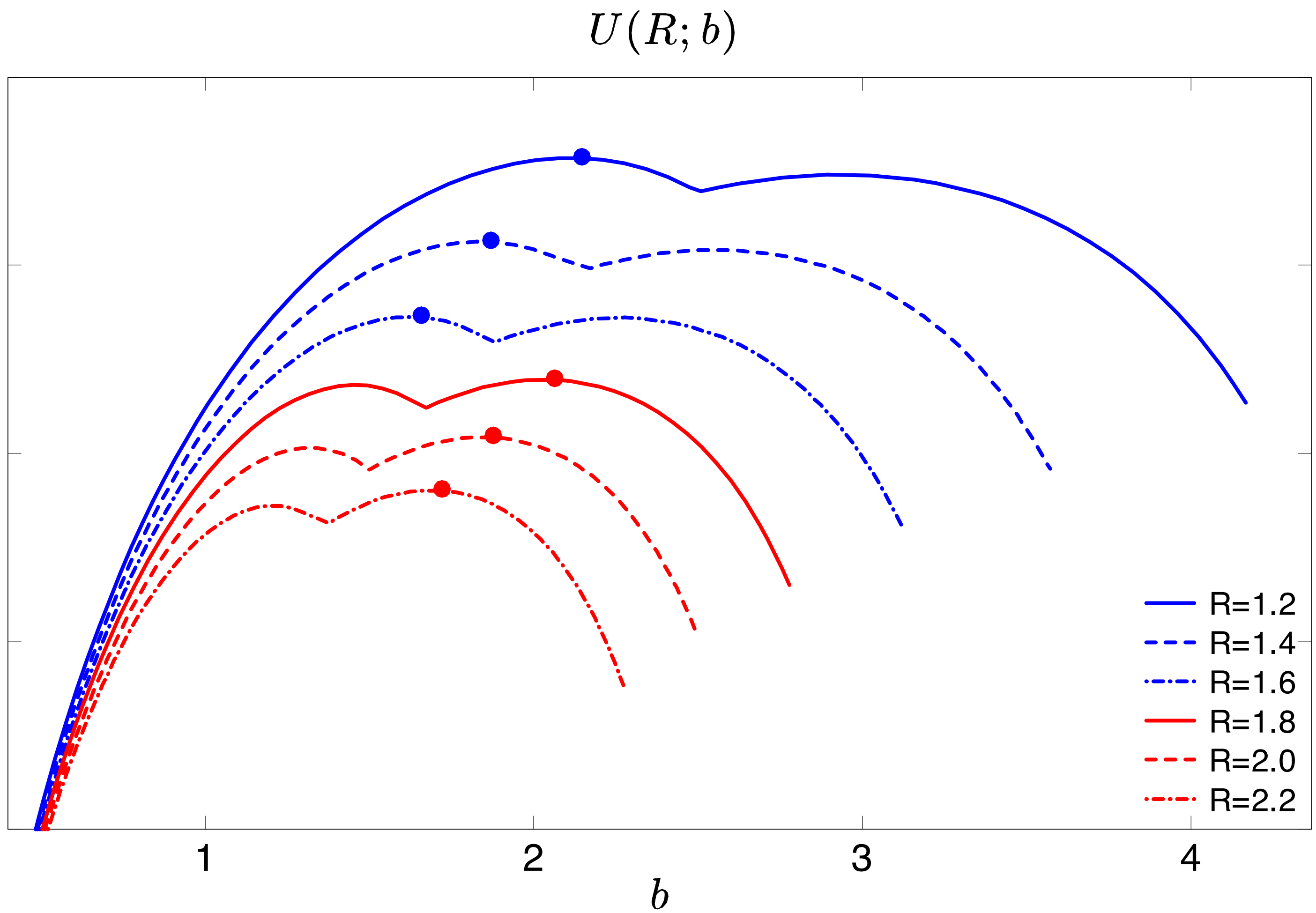
Appendix 1: Demand

Given b, borrower chooses b in order to maximize the objective function

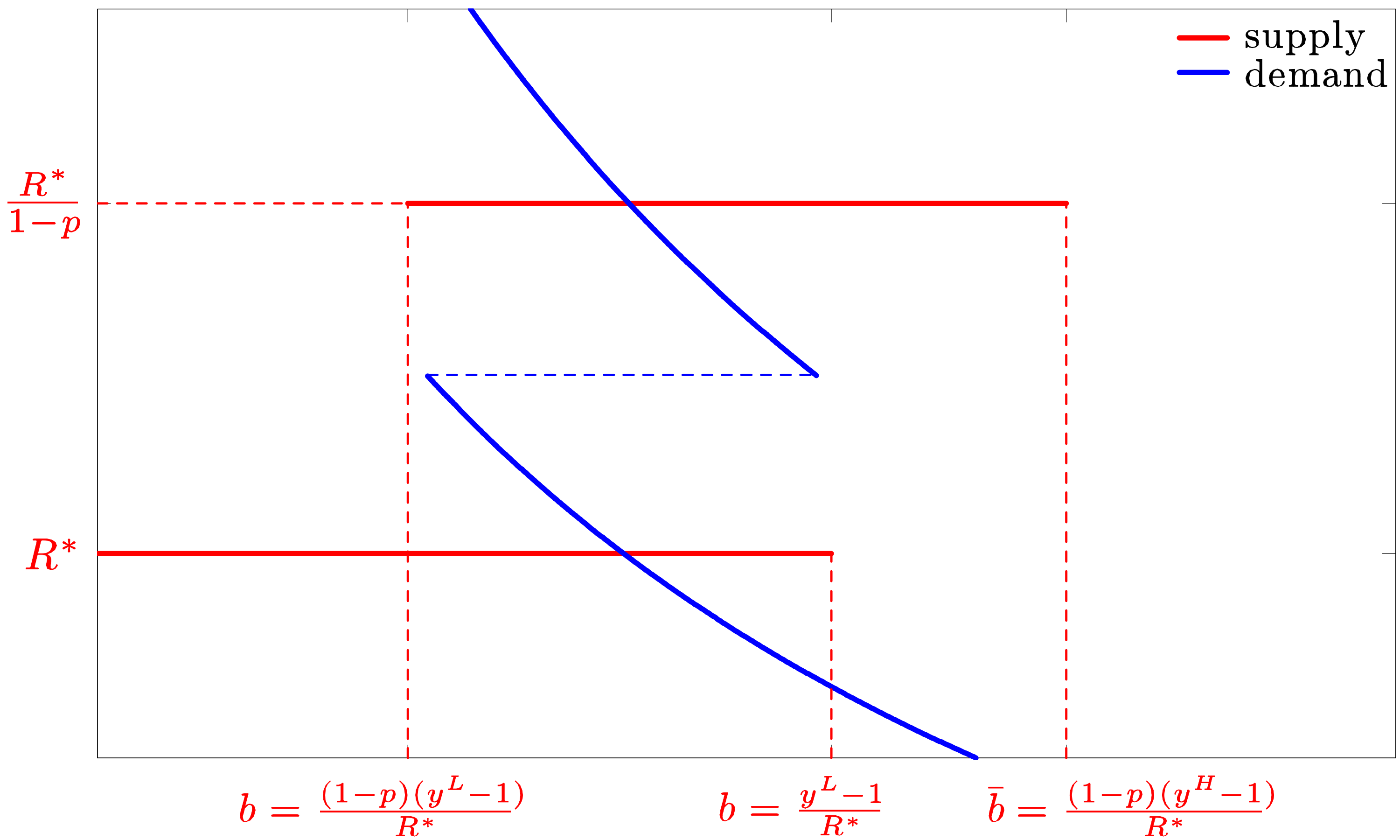
$$W(b) = \begin{cases} \begin{array}{l} U(1+b) + \beta p U\left(y^l - Rb\right) \\ +\beta(1-p)U\left(y^h - Rb\right) \end{array}, & \text{if } b \leq y^l - 1 \\ \\ \begin{array}{l} U(1+b) + \beta p U(1) \\ +\beta(1-p)U\left(y^h - Rb\right) \end{array}, & \text{if } y^l - 1 < b \leq y^h - 1 \\ \\ U(1+b) + \beta U(1), & \text{if } b > y^h - 1 \end{cases}$$



$U(R;b)$
$R = 1.2$
$\underline{b} = \frac{y^l - 1}{R}$
$\bar{b} = \frac{y^h - 1}{R}$



$U(R;b)$
1
2
3
4
b
R=1.2
R=1.4
R=1.6
R=1.8
R=2.0
R=2.2


supply
demand
$\frac{R^*}{1-p}$
R^*
$b = \frac{(1-p)(y^L-1)}{R^*}$
$b = \frac{y^L-1}{R^*}$
$\bar{b} = \frac{(1-p)(y^H-1)}{R^*}$